# Hybrid Beamforming with Blockwise Beam Selection for mmWave Beamspace MIMO systems

Md. Abdul Latif Sarker[1], Dong Seog Han[2]
[1] *Center for ICT & Automotive Convergence*
[2]*School of Electronic and Electrical Engineering, Daegu, South Korea*
Email:{latifsarker, dshan}@knu.ac.kr

***Abstract*—Hybrid beamforming is a signal processing technique composed of multiple components that is of significant importance in reducing transceiver costs and improving spectral efficiency in the context of highly directive wireless communications. Conventional hybrid beamforming leads to considerable baseband computation and suboptimal amplitude factor due to the use of extensive antenna arrays. Hence, we propose a blockwise beam selection (BBS) algorithm to achieve a low-dimensional hybrid beamformer and minimize the computation of baseband processing in mmWave beamspace multiple-input multiple-output lens antenna array systems. In this paper, we first present a block-wise design of a transmitting analog beamformer. In order to achieve a low-dimensional hybrid beamformer, we then apply the proposed BBS algorithm that significantly reduces the dimensions of the baseband processor and enhances the system performance. Finally, the simulation results validate the superiority of the proposed BBS algorithm compared to its traditional non-blockwise beam selection counterparts.**

***Index Terms*—Millimeter wave beamspace MIMO, beamforming, blockwise beam selection, spectral efficiency**

## I. Introduction

Millimeter wave (mmWave) is one of the most promising technologies for next-generation wireless systems, which overcome the spectrum shortage of 4G communications [1]. In contrast, large-scale multiple-input multiple-output (MIMO) achieves high beamforming gain in the mmWave systems [2]–[4]. Due to power and cost constraints, fully digital beamforming solutions are unfeasible and hybrid beamforming becomes more difficult. By setting both the phase and magnitude of baseband signals, it is possible to design a hybrid beamforming scheme in the digital domain. Analog-to-digital converters, digital-to-analog converters and radio-frequency chains are required for this scheme, all of which are costly and energy-intensive. It is difficult for mmWave beamspace MIMO systems to effectively deploy hybrid beamformers because of the poor amplitude factor caused by large antenna arrays, phase angle optimization caused by large beamwidths, channel sparsity, severe path losses, and baseband complexity.

To address the beamformer's design difficulty, researchers investigated several design architectures in [3]–[7]. Hybrid beamforming with a hybrid beam selection (HBS) scheme was proposed to mitigate the rate loss in [3]. In [4], the authors studied a hybrid beamforming scheme with mixed-timescale. The authors proposed a reduced-dimensional subspace codebook based on a single beam selection (SBS) algorithm in [5]. Hybrid beamforming with multiple beam selection (MBS) algorithms was investigated to get a good trade-off between feedback performance and cost in [6]. The authors proposed an iterative hybrid beamforming approach to maximize the weighted sum rate performance in [7]. Particularly, the traditional hybrid beamforming with a non-blockwise beam selection algorithm such as SBS, MBS, and HBS algorithm is widely used in mmWave MIMO communication systems. Owing to the huge baseband complexity with a poor magnitude factor of non-blockwise SBS, MBS, and HBS-based hybrid beamforming, we propose a blockwise beam selection (BBS) algorithm in this letter. According to our knowledge, no research has been conducted on the proposed BBS algorithm.

In this letter, we propose a BBS algorithm to obtain a low-dimensional hybrid beamforming for mmWave beamspace MIMO using lens antenna arrays. The proposed BBS algorithm can reduce the architectural complexity of a transmit analog beamformer as well as a baseband processor. The proposed BBS algorithm also achieves a high amplitude factor, which plays an important role to improve the beamforming gain. We first present a block-wise design of a transmit analog beamformer. In order to achieve the proposed hybrid beamforming, we then consider a blockwise beam selection algorithm that significantly reduces the dimensions of the baseband processor and enhances the spectral efficiency performance. Finally, on the basis of a computer simulation, we validate the per-user spectral efficiency as well as the beamforming gain of the proposed BBS algorithm compared to its traditional non-blockwise beam selection algorithm.

## II. System Model

Consider a downlink communication for mmWave beamspace MIMO using lens antenna arrays as depicted in Fig.1, where $K$ single-antenna users are simultaneously served by the base station (BS) in the system and the BS is equipped with $N_t$ transmitting antennas. The number of RF chains is represented as $N_{RF}$, where $N_t \gg K$ and $K \leq N_{RF} \leq N_t$. Now, we take a downlink mmWave $N_t \times 1$ channel vector $\mathbf{h}_k = \sqrt{\frac{N_t}{L}} \sum_{l=1}^{L} \alpha_{k,l}\mathbf{a}(\phi_{k,l})$, where $\alpha_{k,l}$ denotes the complex path gain at the $l$-th path and the $k$-th user, $L$ represents the number of channel path, $\mathbf{a}(\phi_{k,l})$ represents the $N_t \times 1$ steering vector of the $l$-th path and the $k$-th user, the $N_t$-element uniform linear array (ULA) with $\phi_{k,l} = \frac{d}{\lambda}\sin(\theta_{k,l})$ is treated as in [1], [3], where $c$ denotes the

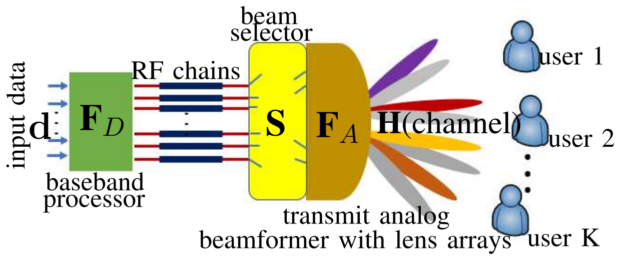

Fig. 1: System model is represented by a beamspace MIMO using lens antenna arrays.

light speed, $\lambda = c/f_c$ is the wavelength, $f_c$ defines the carrier frequency, $d = c/2f_c$ represents the antenna spacing, and $\theta_{k,l}$ is the angle of departure (AoD) of the $l$-th path at $k$-th user. The received signal, $y_k$, can therefore be expressed as

$$y_k = \mathbf{h}_k^H \mathbf{F}_A \mathbf{S} \mathbf{F}_D \mathbf{d} + n_k, \tag{1}$$

where $\mathbf{F}_A$ is the $N_t \times N_t$ spatial-domain discrete Fourier transform matrix [3], [5], [8], $\mathbf{S}$ is a $N_t \times N_{RF}$ beam selection matrix, $\mathbf{F}_D \in \mathbb{C}^{N_{RF} \times K}$ denotes the zero-forcing (ZF) baseband processing matrix and $\mathbf{F}_D$ satisfy $||\mathbf{F_A}\mathbf{F}_D||_F^2 = K$ to meet total transmit power constraint, $\mathbf{d} \in \mathbb{C}^{K\times 1}$ is a transmitted data vector that satisfies $\mathbb{E}[\mathbf{d}\mathbf{d}^H] = \frac{P}{K}\mathbf{I}_K$, $\mathbb{E}[\cdot]$ is called the expectation operator, the operator $(\cdot)^H$ denotes Hermitian, $P$ denotes the transmit power at the BS and $n_k \sim \mathcal{CN}(0,1)$ represents the additive white Gaussian noise with zero-mean and unit variance.

From (1), the system can achieve the following spectral efficiency at the $k$-th user:

$$R_k = \log_2 \{1 + \gamma_k\}, \tag{2}$$

where $\gamma_k$ represents the received signal-to-interference-plus-noise ratio at the $k$-th user is given by

$$\gamma_k = \frac{\frac{P}{K}|\mathbf{h}_{eq,k}\mathbf{f}_{D,k}|^2}{1 + \frac{P}{K}\sum_{j=1, j\neq k}^{K} |\mathbf{h}_{eq,k}\mathbf{f}_{D,j}|^2}, \tag{3}$$

where $\mathbf{h}_{eq,k} = \mathbf{h}_k^H \mathbf{F}_A \mathbf{S}$ and $\mathbf{f}_{D,k}$ denotes the $N_{RF} \times 1$ vector of baseband processing matrix $\mathbf{F}_D$.

## III. Proposed BBS Algorithm

This section demonstrates a blockwise beam selection (BBS) algorithm to obtain a low-dimensional baseband processor for mmWave beamspace MIMO systems. Using the proposed BBS algorithm, we redraw the performance metric and compute the achievable spectral efficiency as follows.

Let $\bar{N}_{RF} = N_{RF}/2$, where $K \leq \bar{N}_{RF} < N_{RF}$, and $N \gg K$. For obtaining a low-dimensional channel matrix from $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K]^H$, we consider a $K \times 2N$ matrix $\mathbf{H}^b = [\mathbf{H}^o \ \mathbf{H}^e]$, where $\mathbf{H}^o \in \mathcal{C}^{K\times N}$ denotes the block channel matrix for an odd number of antenna elements and $\mathbf{H}^e \in \mathcal{C}^{K\times N}$ denotes the block channel matrix for an even number of antenna elements. Hence, we obtain a $K \times 2N$ equivalent channel $\tilde{\mathbf{H}}^b$, which is given using a block diagonal transmit beamformer $\mathbf{F}_A^b$ and $\mathbf{H}^b$ as $\tilde{\mathbf{H}}^b = \mathbf{H}^b\mathbf{F}_A^b = [\tilde{\mathbf{H}}^o \ \tilde{\mathbf{H}}^e]$, where $\tilde{\mathbf{H}}^o = \mathbf{H}^o\mathbf{F}_A^o$ and $\tilde{\mathbf{H}}^e = \mathbf{H}^e\mathbf{F}_A^e$. For obtaining a blockwise beam selection design, we consider a $2N \times \bar{N}_{RF}$ beam selection matrix $\mathbf{S}^b$ as $\mathbf{S}^b = [\mathbf{S}^o; \ \mathbf{S}^e]$, where selector $\mathbf{S}^o$ denotes an $N \times \bar{N}_{RF}$ matrix for selecting an odd number of RF chains and selector $\mathbf{S}^e$ denotes an $N \times \bar{N}_{RF}$ matrix for selecting an even number of RF chains. The selector $\mathbf{S}^o$ and $\mathbf{S}^e$ comprises one non-zero element '1' in each column with different positions. The blockwise optimal beam is selected by

$$b_1^* : \underset{b_1 \in \{2\xi - 1\}}{\arg\max} \ ||[\tilde{\mathbf{H}}^o]_{b_1}||_2^2; \ \ \mathcal{S}_1^* = \{b_1^*\} \tag{4}$$

and

$$b_2^* : \underset{b_2 \in \{2\xi - 2\}}{\arg\max} \ ||[\tilde{\mathbf{H}}^e]_{b_2}||_2^2; \ \ \mathcal{S}_2^* = \{b_2^*\}, \tag{5}$$

where $\xi = 1 : N$. We now identify its strongest beam and generate a blockwise beam index set as follows:

$$\mathcal{I}_1 = \{\mathcal{S}_1^*, b_1^*\}, \ \ \mathcal{I}_2 = \{\mathcal{S}_2^*, b_2^*\}. \tag{6}$$

Based on (4)-(6), the blockwise beam selection procedure is summarized in **Algorithm 1**.

**Algorithm 1** Proposed BBS Algorithm

1: **Input parameters:** $\mathbf{H}^o$, $\mathbf{H}^e$, $\mathbf{F}_A^o$, $\mathbf{F}_A^e$, $K$, $\bar{N}_{RF}$, and $N$.
2: **Output:** $\mathcal{S}_1^*$ and $\mathcal{S}_2^*$.
3: Solve (4) for initialization $\mathcal{S}_1^*$.
4: Construct the beam index set $\mathcal{I}_1$ as per (6).
5: **for** $\nu_1 = 1 : 1 : \bar{N}_{RF}$ **do**
   $\mathbf{S}^o(\mathcal{I}_1(\nu_1), \nu_1) = 1,$
6: **end for**
7: Solve (5) for initialization $\mathcal{S}_2^*$.
8: Construct the beam index set $\mathcal{I}_2$ as per (6).
9: **for** $\nu_2 = 1 : 1 : \bar{N}_{RF}$ **do**
   $\mathbf{S}^e(\mathcal{I}_2(\nu_2), \nu_2) = 1,$
10: **end for**
11: Obtain $\mathcal{S}_1^*$ and $\mathcal{S}_2^*$ according to (4) and (5).

Following **Algorithm 1**, a multi-stream baseband processor can be used to $\mathbf{H}_{eq}^b$, where a blockwise $\bar{N}_{RF} \times K$ dimensional zero-forcing (ZF) matrix is accomplished by the constructed equivalent channel $\mathbf{H}_{eq}^b$ as follows:

$$\mathbf{F}_D^b = (\mathbf{H}_{eq}^b)^H \{\mathbf{H}_{eq}^b (\mathbf{H}_{eq}^b)^H\}^{-1} \mathbf{\Lambda}^b, \tag{7}$$

where $\mathbf{\Lambda}^b$ is the diagonal matrix, introduced to the normalized transmit power, $\mathbf{H}_{eq}^b = [\mathbf{H}_{eq}^o \, \mathbf{H}_{eq}^e]$, $\mathbf{H}_{eq}^o = \mathbf{H}^o\mathbf{F}_A^o\mathbf{S}^o$ is the $K \times \bar{N}_{RF}$ equivalent channel matrix for odd number of antenna elements, $\mathbf{H}_{eq}^e = \mathbf{H}^e\mathbf{F}_A^e\mathbf{S}^e$ is the $K \times \bar{N}_{RF}$ equivalent channel matrix for even number of antenna elements.

Setting (7) in (2), we can measure the achievable spectral efficiency at $k$-th user given as $R_k = \log_2\{1 + \gamma_k^b\}$, where

$\gamma_k^b$ denotes the blockwise estimation of signal-to-interference plus noise ratio, that is

$$\gamma_k^b = \frac{\frac{P}{K}|\mathbf{h}_{eq,k}^b \mathbf{f}_{D,k}^b|^2}{1 + \frac{P}{K}\sum_{\bar{j}=1,\bar{j}\neq k}^{K} |\mathbf{h}_{eq,k}^b \mathbf{f}_{D,\bar{j}}^b|^2}, \tag{8}$$

where $\mathbf{h}_{eq,k}^b$ denotes the vector of $\mathbf{H}_{eq}^b$ and $\mathbf{f}_{D,k}^b$ represents the vector of $\mathbf{F}_D^b$.

## IV. Simulation Results and Discussion

In order to demonstrate the effectiveness of the proposed BBS algorithm versus the traditional non-blockwise beam selection algorithm, we provide simulation results in this section. The received signal-to-noise-ratio (SNR) is set out as $\gamma = 10\log_{10}\frac{P}{K}\mathbb{E}[||\mathbf{h}_{eq}^b||^2]$ for all users. For the line-of-sight points of the mmWave channel, we measure $\alpha_{k,l} \sim \mathcal{CN}(0, 10^{-0.1PL})$, where the path-loss $PL$ is defined in [9]. The azimuth angle of departures is randomly sorted out from the uniformly distributed in the interval $(-\pi/2, \pi/2)$. The frequency range and wavelength are used as 28 GHz and $\lambda = 0.01$ m, respectively.

In a fair comparison, the complexity of the proposed BBS algorithm is about $\mathcal{O}(K\bar{N}_{RF}N^2)$, which outperformed the non-blockwise beam selection algorithm such as the traditional SBS algorithm [5], [8] and the HBS algorithm [3], as shown in TABLE I. Fig. 2 illustrates a comparison of the proposed BBS algorithm's spectral efficiency per user. In computer simulation, the parameters are used as $N_t = 256$, $N_{RF} = 24$, $K = 8$, $N = 128$, $\bar{N}_{RF} = 12$, $L = 3$. All curves were also averaged using 1000 independent channel realizations. At 30 dB SNR, the user spectral efficiency of the proposed BBS algorithm is almost 4.96 bps/Hz. Conventional HBS and SBS algorithms exhibited per-user spectral efficiency of 3.56 bps/Hz and 3.09 bps/Hz, respectively. It is estimated that the proposed BBS algorithm increases the per-user spectral efficiency by around 1.39 per Hz. Compared with the SBS algorithm, the HBS algorithm shows better spectral efficiency performance in Fig. 2, in contrast, due to the large-dimensional beam selector, the HBS algorithm has a higher computational complexity than the SBS algorithm (see TABLE I).

## V. Conclusion

We have proposed a BBS algorithm that obtains a low-dimensional hybrid beamformer and outperforms the traditional non-blockwise beam selection algorithm in this letter. Compared to the non-blockwise beam selection algorithm, the proposed BBS algorithm generates 1.41 times more amplitude factor. We have corroborated the effectiveness of the spectral efficiency using the analytical results and compared them with the traditional non-blockwise SBS and HBS algorithms. The traditional SBS and HBS algorithms exhibited a worse system performance when compared with the proposed BBS algorithm. Future studies will explore the application of the proposed BBS algorithm to multi-carrier communications in the terahertz band.

TABLE I: Comparison of the Computational Complexity

| Algorithms | Complexity |
|---|---|
| SBS algorithm [5], [8] | $\mathcal{O}(KN_{RF}N_t^2)$ |
| HBS algorithm [3] | $\mathcal{O}(K(N_{RF_{g1}} + N_{RF_{g2}})N_t^2)$ |
| Proposed BBS algorithm | $\mathcal{O}(K\bar{N}_{RF}N^2)$ |

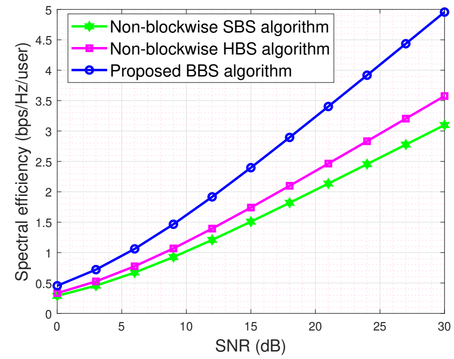

Fig. 2: Spectral efficiency in bps/Hz/user.

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