Performance Analysis of the V2V System Based on the *Nakagami-m* Fading Channel

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***Abstract*—With the rapid advancement of the automotive industry and artificial intelligence,automobiles have brought unparalleled convenience and efficiency to people's lives. However, they have also introduced certain challenges, such as traffic congestion.The technology of Vehicular Ad-Hoc Networks (*VANETs*) stands as a pivotal element of the Intelligent Transportation System(ITS) and has garnered widespread attention from researchers worldwide. *VANETs* primarily focus on enabling seamless information exchange among vehicles, laying the groundwork for reliable technical support to achieve autonomous driving on roads. Existing literature has extensively examined the outage probability and average bit error rate (*BER*) in wireless digital systems operating under *Nakagami-m* fading channels. Most of these analyses have been confined to static wireless terminals, where the received signal power follows a gamma distribution. Nevertheless,real-world scenarios often involve networks with mobile receiving nodes, static wireless terminals do not satisfy the mobility of transceiver nodes,research on mobile transceiver nodes is necessary.In this paper, we delve into three distinct wireless network deployment topologies, employing the Random Waypoint (RWP) mobility model. In such dynamic systems, the received power does not conform to a gamma distribution in the presence of *Nakagami-m* fading. We derive the probability density function (PDF) and cumulative distribution function (CDF) of the received signal power for mobile nodes. Consequently, we determine the average *BER* for general modulation schemes, considering different m_s values and three diverse topology structures' impact on the average *BER* of *Nakagami-m* fading channels.**

***Keywords*—*Nakagami-m* fading, *VANETs*, *BER* ,Vehicles to Vehicle communicate**

I. Introduction

With the continuous progress of society and economy, the number of automobiles has been steadily increasing. While cars serve as the primary means of transportation, bringing convenience to people's lives, they also present a multitude of challenges such as urban congestion, traffic safety issues, and energy consumption concerns. In response to these challenges, *VANETs* have emerged as a wireless communication technology based on Vehicle-to-Vehicle (V2V) interactions, enabling information exchange and collaboration among vehicles.

In *VANETs*, V2V communication is influenced by various factors, including channel fading, multipath effects, and interference. Among these, channel fading stands as a crucial issue in *VANETs*, as it can lead to decreased communication quality and data transmission rates. Literature[1] investigated the variation of propagation loss in vehicle-to-vehicle channel in highway, urban, suburban, and rural environments based on narrowband channel measurements at 5.2GHZ carrier frequency and proposed a path loss prediction model in various scenarios. Literature[2] carried out a large number of vehicle-to-vehicle narrowband channel measurements in the 5.9 GH band, proposed a double-slope path loss model for suburban scenarios, found that the small-scale fading of the channel in vehicle-to-vehicle scenarios obeys the *Nakagami-m* distribution.

The *Nakagami-m* fading channel is a common fading model used to describe wireless channel characteristics. Within the context of *VANETs*, *Nakagami-m* fading is employed to depict communication quality between vehicles and evaluate communication system performance. However, due to the complexity and variability of the mobile communication environment, researching the performance of mobile communication systems remains challenging. Hence, designing and evaluating the performance of mobile communication systems have become crucial for their practical applications. In the *Nakagami-m* fading channel, the instantaneous received signal power of a wireless communication system follows a gamma distribution [3].

Previous research on the performance of wireless communication systems under *Nakagami-m* fading has predominantly focused on analyzing the performance of static wireless networks. Literature[4] investigated the path loss delay distribution, and Doppler characteristics of vehicle-to-vehicle channels in highways based on 5.2GHz band MIMO measurements. Literature[5] investigated the vehicle-to-vehicle narrowband channel propagation loss in highway and rural scenarios in the 5.9 GH band and proposed a segmented linear path loss prediction model. Literature[6] shows that an L-branch maximal ratio combining (MRC) receiver with an average signal-to-noise ratio (SNR) in *Nakagami-m* fading channels is equivalent to an mL-branch MRC receiver with average SNR in Rayleigh fading channels in terms of symbol error probability (SEP) when mL is an integer. Literature[7] studies the secrecy outage probability of diversity schemes over arbitrary *Nakagami-m* fading channels. Literature[8] performance analysis of multi-hop networks over independent but non-identically distributed *Nakagami-m* fading channels for half-duplex (HD) and decodeand- forward (DF) transmission protocols is revised. Where both the receiver and transmitter remain stationary. Scenarios involving both mobile receivers and transmitters have not been thoroughly considered. The widely used mobility model for analysis is the RWP, initially proposed by Johnson and Maltz[9]. The RWP mobility model results in an asymptotically non-uniform spatial node distribution, which is beneficial for performance analysis.

The *BER* is a metric used to measure the accuracy of data transmission within a specified time period. It represents the ratio of erroneous bits to the total transmitted bits. The *BER* analysis under specific conditions is essential for improving the performance of wireless communication systems and enhancing data transmission quality. The main contributions of this paper are as follows:

- This paper utilizes the RWP mobility model to describe the mobility of transceivers and incorporates the MRC diversity technique to overcome multipath fading, thereby conducting an in-depth analysis of the received power in *Nakagami-m* fading channels.
- The PDF and CDF of the received power in *Nakagami-m* fading channels under the RWP mobility model are derived and presented.
- Furthermore, the study explores the influence of three different topology structures and various m_s values on the *BER* performance in *Nakagami-m* fading channels.

The subsequent sections of this paper are as follows: Section II introduces the signal model for *Nakagami-m* fading channels and the RWP mobility model, along with the derivation of PDF and CDF for the received signal,a derivation of *BER* is then given. Section III provides simulation results based on the derived expressions. Lastly, Section IV concludes the paper.

II. SYSTEM MODEL

A. Signal Model

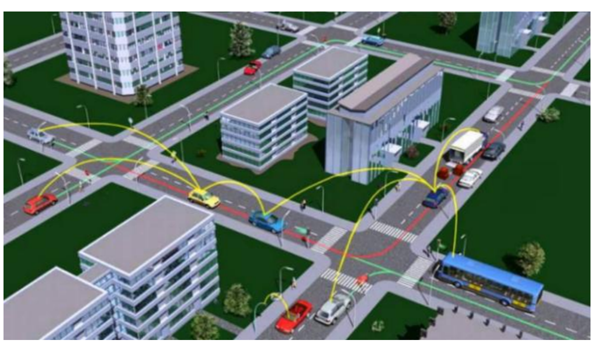

Fig. 1. Urban V2V scenarios

Figure 1 illustrates a scenario of V2V communication in an urban roadway, where vehicles are dynamically communicating with surrounding vehicles in an intelligent manner.

As widely acknowledged, in a stationary *Nakagami-m* fading channel, the envelope of the received signal power is expressed by equation (1) [10].

$$f\left(r\right)=2\left(\frac{m_s}{\Omega_s}\right)^{m_s}\frac{r^{2m_s-1}}{\Gamma(m_s)}\times exp\left(-\frac{m_s}{\Omega_s}r^2\right) \tag{1}$$

Where m_s is the fading parameter that determines the type of fading channel, Ω_s is the average received power. The Gamma Function $\Gamma(a)=\int_0^\infty t^{a-1}e^{-t}dt$.

MRC is the optimal choice in diversity combining technology. Compared with selection combining and equal gain combining, it can achieve the best performance. Its performance improvement comes from higher signal-to-noise ratio brought by array gain which brings better *BER* characteristics.Taking into account an N-branch MRC reception system, the PDF of the received signal power is expressed as equation(2) [11].

$$f\left(r\right)=\frac{2m_s^{m_s}}{\Gamma(m_s)\Gamma(N)}\frac{r^{2m_s+N-2}}{\Omega_s^{m_sN}}\times exp\left(-\frac{m_s}{\Omega_s}r^2\right) \tag{2}$$

In the equation(2), N represents the topology array for the N-branch MRC reception system. Equations (1) and (2) describe the PDF of the received power for static nodes. However, for mobile nodes, $\Omega_s=P_ty^{-\alpha}$ represents the average received power, which varies with the transmission distance, where P_t is the transmission power, and y is the transmission distance. The parameter α ($2\leq\alpha\leq5$) denotes the path loss exponent, which is dependent on the characteristics of the transmission environment.

In the wireless transmission network with mobile nodes, the transmission power remains constant, while the propagation distance y is a random variable. Consequently, the conditional PDF of the received signal power, given a specific transmission distance y, can be expressed as follows:

$$f_r(r\mid y)=\frac{2m_s^{m_s}}{\Gamma(m_s)\Gamma(N)}\frac{r^{2m_s+N-2}}{(P_ty^{-\alpha})^{m_sN}}\times exp\left(-\frac{m_s}{P_ty^{-\alpha}}r^2\right) \tag{3}$$

B. Mobile Model

Assuming that the range of values of the propagation distance y between the vehicle and the vehicle moving in the urban scene is from 0 to D ($0\leq y\leq D$), the PDF of the received power can be expressed as follows:

$$f_r\left(r\right)=\int_0^D f_r\left(r\mid y\right)f_y\left(y\right)\mathrm{d}y \tag{4}$$

In mobile environments, there are various mobility models available to describe the distribution of distances between transmitters and receivers. These models assume that nodes move randomly and can be classified based on their statistical characteristics. As is well known, the RWP model often results in a non-uniform spatial distribution of network nodes [12], whereas the Random Direction (RD) mobility model assumes a uniform spatial distribution [13]. Both RD and RWP models can represent the steady-state spatial node distribution as a polynomial of the transmission-reception distance denoted as

r [14], [7]. For this study, we focus on the RWP model, but the analysis can be extended to any rotationally symmetric spatial distribution that can be approximated by a polynomial function of the transmission-reception distance.

In the RWP migration model, it is usually assumed that the receiving vehicle is located at randomly selected coordinate points within the service area, depending on the network topology. For a one-dimensional (1-D) topology, we consider lines where the access vehicle is located at the origin. The two-dimensional (2-D) topology is assumed to be circular, while the three-dimensional (3-D) topology is a spherical network. In both the 2-D and 3-D net-terminal topologies, it is assumed that the access vehicle is located at the origin.

The steady-state spatial node distribution of the RWP mobility model is polynomial in the send and receive distance r. The PDF of the distance r is given by the general form[6].

$$f_r(r)=\sum_{i=1}^{n}B_i\frac{r^{\beta_i}}{D\beta_i+1},0\le r\le D \tag{5}$$

In this context, the parameters n, B_i, and β_i are dependent on the dimensions considered in the topology and can be summarized as follows [14]. For example, according to [14, Table 1], for a 1-D topology, when n=2, B_i=[6, -6], and β_i=[1, 2]; for a 2-D topology, when n=3, B_i=(1/73)*[324, -420, 96], and β_i=[1, 3, 5]; and for a 3-D topology, when n=3, B_i=(1/72)*[735, -1190, 455], and β_i=[2, 4, 6].

In a one-dimensional (1-D) topology, the PDF expression for its received vehicle power is:

$$\begin{aligned}f_{RWP}(r)&=\int_0^1\frac{2m_s^{m_s}}{\Gamma(m_s)\Gamma(N)}\times\frac{r^{2m_s+N-2}}{(P_ty^{-\alpha})^{m_sN}}\\&\times\exp(-\frac{m_s}{P_ty^{-\alpha}}r^2)\times(6y-6y^2)dy\\&=\frac{2m_s^{m_s}}{\Gamma(m_s)\Gamma(N)}\times\frac{r^{2m_s+N-2}}{P_t^{m_sN}}\\&\int_0^1y^{\alpha m_sN}\times\exp(-\frac{m_s}{P_ty^{-\alpha}}r^2)\times(6y-6y^2)dy\\&=\frac{12m_s^{m_s}}{\Gamma(m_s)\Gamma(N)}\times\frac{r^{2m_s+N-2}}{\alpha P_t^{m_sN}}\\&\int_0^1(y^{\alpha(m_sN+\frac{2}{\alpha}-1)}-y^{\alpha(m_sN+\frac{3}{\alpha}-1)})\times\exp(-\frac{m_sr^2}{P_ty})dy\\&=\frac{12m_s^{m_s}}{\Gamma(m_s)\Gamma(N)}\times\frac{r^{2m_s+N-2}}{\alpha P_t^{m_sN}}\\&[\Gamma(\frac{2}{\alpha}+m_sN,\frac{m_sr^2}{P_t})-\Gamma(\frac{3}{\alpha}+m_sN,\frac{m_sr^2}{P_t})]\end{aligned} \tag{6}$$

Integrating the PDF of the vehicle's received power gives the CDF of the vehicle's received power, the CDF expression for its received vehicle power is:

$$\begin{aligned}F_{RWP}(r)=\int_0^1&\frac{12m_s^{m_s}}{\Gamma(m_s)\Gamma(N)}\times\frac{r^{2m_s+N-2}}{\alpha P_t^{m_sN}}\times\\&[\Gamma(\frac{2}{\alpha}+m_sN,\frac{m_sr^2}{P_t})-\Gamma(\frac{3}{\alpha}+m_sN,\frac{m_sr^2}{P_t})]\end{aligned} \tag{7}$$

Where $\Gamma(z_1,z_2)$ is the lower incomplete gamma function.

In the digital transmission of V2V systems , *BER* is the number of error codes transmitted in the channel divided by the total number of codes transmitted in the channel. [7]From the definition of *BER* it can be derived that the system average *BER* of binary modulation with respect to the probability density function is given by:

$$\begin{aligned}\overline{P}_b&=\int_0^\infty\frac{\Gamma(b,ar)}{2\Gamma(b)}f_{RWP}(r)dr\\&=\frac{a^b}{2\Gamma(b)}\int_0^\infty x^{b-1}e^{-ar}F_{RWP}(r)dr\end{aligned} \tag{8}$$

where $\Gamma(z_1,z_2)$ is the upper incomplete gamma function[[15], eq.(8.350.2)] and $\Gamma(z)$ is gamma function[[15], eq.(8.310.1)]. The parameters a, b∈ (1/2, 1) depend on the type of binary modulation/demodulation employed [16].

III. SIMULATION RESULT

In this section, we explore the influence of varying the diversity factor N on the PDF of the output in *Nakagami-m* fading channels, utilizing diversity techniques. Additionally, we investigate the impact of changing m_s on the *BER*. The section further presents *BER* results for three different RWP mobility models in 1-D, 2-D, and 3-D scenarios.

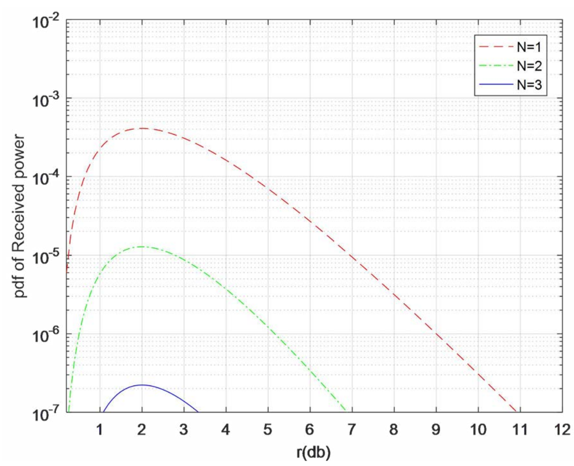

Fig. 2. Illustrates the PDF of the output in a *Nakagami-m* fading channel.

Figure 1 illustrates the output PDF of a *Nakagami-m* fading channel under the RWP mobility model. The graph showcases the cases for N=1, N=2, and N=3,from the PDF of the received power of the fading channel Eq. (6), the PDF of the received power increases with the increase of N for the same SNR, as shown in the figure, and the PDF of the received power is maximum at N=1.

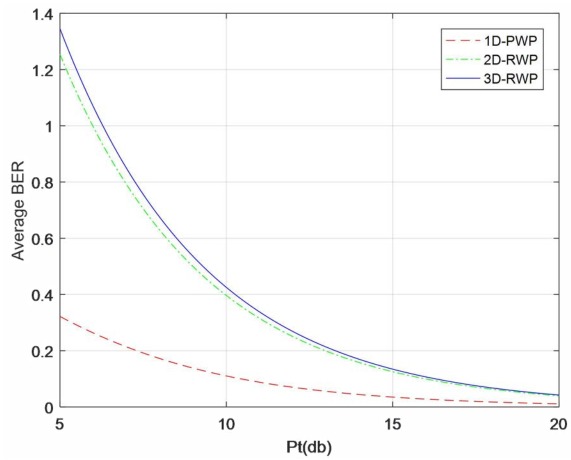

Fig. 3. Influence of different m_s on interrupt probability

Figure 2 illustrates the impact of three different topologies, namely 1-D, 2-D, and 3-D RWP mobility models, on the *BER*. For a 1-D topology, we consider lines where the access vehicle is located at the origin. The 2-D topology is assumed to be circular, while the 3-D topology is a spherical network.The graph clearly demonstrates that the 3-D model yields the highest *BER*, while the 1-D model yields the lowest *BER*.

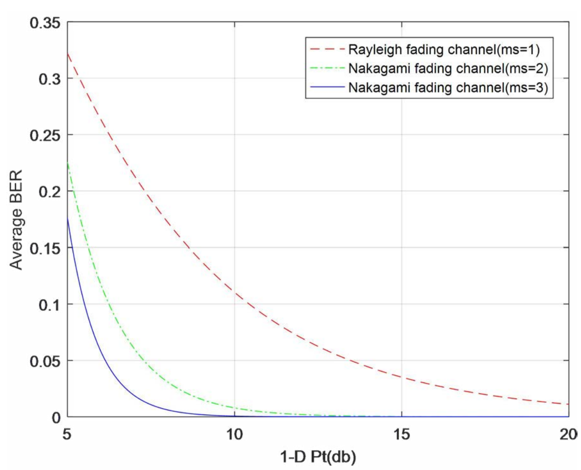

Fig. 4. Influence of different m_s on interrupt probability

Figure 3 displays the impact of different m_s values on the *BER* in a 1-D RWP mobility model. m_s denotes the fading factor, the larger m_s is, the smaller the corresponding channel fading is, when m_s=1, the *Nakagami-m* fading channel is a Rayleigh fading channel, at this time, the channel fading is the largest and the *BER* is the largest, as m_s increases, the channel fading decreases and the *BER* starts to decrease, when m_s=3, the *BER* is the smallest.

IV. Conclusion

This paper analyzes the performance under *Nakagami-m* fading channels, incorporating the MRC centering technique to combat multipath fading. The RWP mobility model applied between the transmitter and receiver is used to analyze the performance in case the transmitter and receiver are mobile. The corresponding mathematical formulas are derived to analyze the PDF and CDF of the received signals under *Nakagami-m* fading channel, the effect of change in m_s on the average *BER*, and the effect of different topologies on the *BER* of *Nakagami-m* fading channel are also analyzed, in which the 3-D model has the largest *BER* and the 1-D model has the smallest *BER*.

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References

[1] Karedal J, Czink N,Paier A, et al. Path Loss Modeling for Vehicle-to-Vehicle Communications[J]. *IEEE Transactions on Vehicular Technology*, 2011, 60(1):323-328.
[2] Cheng L, Henty B E,Stancil D D, et al. Mobile Vehicle-to-Vehicle Narrow-Band ChannelMeasurement and Characterization of the 5.9 GHz Dedicated Short Range Communication(DSRC) Frequency Band [J]. *IEEE Journal on Selected Areas in Communications*, 200725(8):1501-1516.
[3] M. K. Simon and M. S. Alouini, Digital Communication over Fading Channels, 2nd ed. *New York: Wiley*, 2005.
[4] Paier A, Karedal J,Czink N,et al. Car-to-Car Radio Channel Measurements at 5 GHz: PathlossPower-Delay Profile, and Delay-Doppler Spectrum [CJ. *//Proceedings of IEEEISWCS, Trond.heim,Norway,OctoBER 17-19.USA:IEEE*, 2007.224-228.
[5] Cheng L, Henty B E, Bai F, et al. Highway and Rural Propagation Channel Modeling for Vehicle-to-Vehicle Communications at 5.9 GHz (CJ. *//Proceedings of IEEE AP-S, San Diego.USA,July 5-11.USA:IEEE*, 2008.1-4.
[6] Y. G. Kim, N. C. Beaulieu and W. -K. Lee, "SEP Performance Using Equivalence in *Nakagami-m* Fading Channels," in *IEEE Transactions on Vehicular Technology*, vol. 65, no. 5, pp. 3792-3795, May 2016, doi: 10.1109/TVT.2014.2385477.
[7] D. Lee, "Secrecy Outage Probability of MIMO Diversity Schemes Over Integer and Real-Valued *Nakagami-m* Fading Channels," in *IEEE Wireless Communications Letters*, vol. 11, no. 4, pp. 727-731, April 2022, doi: 10.1109/LWC.2022.3141711.
[8] E. Fidan and O. Kucur, "Performance of Decode-and-Forward Multihop Networks over i.n.i.d. *Nakagami-m* Fading Channels," *2020 28th Signal Processing and Communications Applications Conference (SIU), Gaziantep, Turkey*, 2020, pp. 1-4, doi: 10.1109/SIU49456.2020.9302257.
[9] E. Hyytia and J. Virtamo, “Spatial node distribution of the random waypoint mobility model with applications,” *IEEE Trans. Mobile Computing*, vol. 5, no. 6, pp. 680–694, Jun. 2006.
[10] V. A. Aalo and C. Mukasa and G. P. Efthymoglou, “Effect of Mobility on the Outage and *BER* Performances of Digital Transmissions over *Nakagami-m* Fading Channels,” *IEEE Transactions on Vehicular Technology, 2016*, vol. 65, no. 4, pp. 2715-2721.
[11] M. Das and B. Sahu, “Effect of MRC Diversity on Outage Probability in Mobile Networks,” *2019 Global Conference for Advancement in Technology (GCAT)*, 2019, pp. 1-4.
[12] C. Bettstetter,G. Resta, and P Santi,“The node distribution of therandom waypoint mobility model for wireless ad hoc networks.” *IEEE Trans. Mobile Computing*, vol. 2, no. 3, pp. 256-269, Jul. 2003.
[13] P. Nain, D. Towsley, Benyuan Liu and Zhen Liu, "Properties of random direction models," *Proceedings IEEE 24th Annual Joint Conference of the IEEE Computer and Communications Societies*., Miami, FL, USA, 2005, pp. 1897-1907 vol. 3, doi: 10.1109/INFCOM.2005.1498468.
[14] K. Govindan, K. Zeng, and P. Mohapatra, “Probability density of the received power in mobile networks,” *IEEE Trans. Wireless Commun*., vol. 10, no. 11,pp. 3613–3619, Nov. 2011.
[15] I. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series, and Products, 6th ed. *New York: Academic Press*, 2000.
[16] K. Simon and M. Alouini, Digital Communication over Fading Channels. *New York, NY, USA: Wiley*, 2005.