Optimization of collision-free trajectories for advanced air mobility under risk of a non-cooperative intruder

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***Abstract*—Advanced air mobility (AAM) is anticipated to revolutionize transportation and mobility by fully exploiting the three-dimensional airspace of urban and regional areas. One of the main challenges for ensuring AAM safety is to manage a non-cooperative aerial intruder such as illegal drones or a group of birds. This paper proposes an iterative convex optimization framework for trajectory planning of multiple eVTOLs to avoid collision with the aerial intruder. Several avoidance strategies are discussed, and simulation results show the validity of the avoidance capability of the proposed algorithm.**

***Index Terms*—advanced air mobility, trajectory optimization, collision avoidance, non-cooperative intruder, sequential convex programming**

I. INTRODUCTION

Due to the severe increase of ground mobility vehicles within limited space and ground traffic infrastructure, conventional urban and regional ground traffic suffers from severe congestion. In recent years, as an attractive game-changer for mobility innovation, advanced air mobility (AAM) has been the subject of active research and development to overcome traditional ground traffic congestion problem. The expected success of the AAM relies on the technological maturity of eVTOL. eVTOL is an electrically powered manned aircraft that can vertically take-off and land even on buildings. Tens or hundreds of AAM stakeholders worldwide are in a competitive race to achieve the early AAM market initiative, as the AAM market is not yet realized and there is no clear dominating company/country in this new emerging market. According to AAM roadmaps in the U.S., Europe and South Korea, within a decade, tens or hundreds of eVTOLs with onboard passengers are to be flying simultaneously in national urban and regional airspace. The aviation safety of the multiple simultaneous eVTOLs should be guaranteed to ensure this optimistic air mobility vision.

As one of the important AAM safety considerations, this paper focuses on the risk of aerial collision and avoidance maneuvers. Many studies have dealt with aerial collision avoidance problems. [1] studied distributed trajectory planner for high-density UAM with cooperative and non-cooperative collision avoidance. [2] proposed a 3D decentralized and asynchronous trajectory planner for tens of aerial vehicles with dynamic obstacles to avoid collision. [3] studied conflict-free 4D UAM flight path planning with consideration of airspace occupancy. A key challenge in AAM is the management of non-cooperative intruders - unidentified aerial vehicles (such as unlicensed drones and eVTOLs) or a group of birds that could be a potential threat to the safe operation of the eVTOLs in the AAM corridor. This paper proposes an iterative optimization framework for multi-eVTOLs flight trajectories in the AAM corridor to avoid the risk of collision with a pop-up intruder. This study aims to provide a simple and straightforward sub-optimal solution approach to simulate various AAM operation scenarios with a pop-up intruder threat, suggest efficient avoidance strategies, evaluate their performance, and finally help AAM stakeholders to design appropriate AAM operational rules in the corridor.

II. SYSTEM MODEL

A. Problem Description

For each country, the concept of operation for AAM is still undergoing further improvement. In this paper, we suppose the following scenario for ease of analysis.

- A corridor with reference flight trajectory is given to facilitate AAM between two vertiports in urban and regional areas. eVTOLs are expected to fly through a predetermined reference flight trajectory in the corridor.
- All eVTOLs in the corridor keep their flight speed as V (m/s) and mutual separation distance D (m) for aviation safety. V and D are pre-determined by the aviation regulatory authority.

This work was supported by Electronics and Telecommunications Research Institute (ETRI) grant funded by the Korean government. [23ZS1200, Fundamental Technology Research for Human-Centric Autonomous Intelligent Systems

- eVTOLs have 3D mobility in the air. However, their mobility is constrained by vehicle flight dynamics defined as velocity limit and acceleration limit.
- A non-cooperative intruder such as illegal aerial vehicles or a group of birds is detected by a ground surveillance system, onboard detection sensors (radar, lidar or vision camera), or V2V communication by other eVTOLs in the same corridor.
- The intruder is expected to pass the AAM corridor, causing a risk of collision. It is assumed that the predicted trajectory of the intruder is known as $\mathbf{p}_{INT}(t)$.
- Each eVTOL should keep safety distance D_{INT} against the intruder.
- When the intruder is detected, time is set as $t = 0$, and the initial position of each eVTOL i is $\mathbf{p}_i^0 = [x_i^0, y_i^0, z_i^0]^T$.
- The coordinate system is set as the following: (+x) as the forward direction in the corridor flight path, (+y) as the lateral left direction, and (+z) as an upward altitude.

When the non-cooperative intruder is detected, the fleet of eVTOLs needs to achieve the following objectives to ensure passenger safety:

- avoiding the intruder by ensuring the required safety distance with the intruder
- keeping separation distance with other eVTOLs to avoid mutual collision
- minimizing excessive eVTOL maneuvers for passenger comfort
- minimizing travel time loss (minimizing arrival time delay)

When the intruder is detected, each eVTOL needs to change its flight trajectory to avoid collision with the intruder. When the eVTOL trajectories are changed, it is inevitable to face schedule delay to the destination vertiport, which is an uncomfortable situation for the passengers.

B. Formulation

In this paper, we aim to minimize the schedule delay of the estimated time of arrival induced by the avoidance/separation maneuver of the eVTOLs and to minimize severe eVTOL maneuvers by vehicle acceleration.

$$\min_{\mathbf{p},\mathbf{v},\mathbf{a}} t_f + w \sum_{i,k} ||\mathbf{a}_i[k]|| \tag{1}$$

subject to

$$\begin{aligned}
\mathbf{p}_i[0] &= \mathbf{p}_i^0;\ \ \mathbf{p}_i[T] = \mathbf{p}_i^f && \forall i && \text{(2a)}\\
\mathbf{v}_i[0] &= \mathbf{v}_i[T] = [V, 0, 0]^T && \forall i && \text{(2b)}\\
\mathbf{p}_i[k+1] &= \mathbf{p}_i[k] + \mathbf{v}_i[k]\Delta t && \forall i && \text{(2c)}\\
\mathbf{v}_i[k+1] &= \mathbf{v}_i[k] + \mathbf{a}_i[k]\Delta t && \forall i && \text{(2d)}\\
||\mathbf{v}_i[k]|| &\leq v_{max};\ \ ||\mathbf{a}_i[k]|| \leq a_{max} && \forall k, \forall i && \text{(2e)}\\
D &\leq ||\mathbf{p}_i[k] - \mathbf{p}_j[k]|| && \forall k, \forall i, j \neq i && \text{(2f)}\\
D_{INT} &\leq ||\mathbf{p}_i[k] - \mathbf{p}_{INT}[k]|| && \forall k, \forall i, j \neq i && \text{(2g)}
\end{aligned}$$

There are N eVTOLs and the optimization problem is to find trajectories of each i-th eVTOL composed of 3D positions $\mathbf{p} = \{\mathbf{p}_i[k]\}$, velocities $\mathbf{v} = \{\mathbf{v}_i[k]\}$ and accelerations $\mathbf{a} = \{\mathbf{a}_i[k]\}$, ensuring collision avoidance with the intruder. The eVTOL index is $i \in \{1, 2, ..., N\}$ and k is a discrete time step. The objective function in (1) aims to minimize the final flight time t_f to reduce travel time delay, and to minimize acceleration cost $\sum_{i,k} ||\mathbf{a}_i[k]||$ to prevent excessive movement. Eqs.(2a)-(2b) define initial/final positions and velocities. Eqs.(2c)-(2e) define dynamics constraints of the eVTOLs (velocity and acceleration limit). Eq.(2f) represents mutual safe separation distance constraints among eVTOLs. Eq.(2g) is to ensure a collision-free safety distance between eVTOL and the intruder.

Note that the trajectory optimization formulation has varying final time (t_f) that is not fixed. Then, the upper bound of the discrete time step k is not known. Also, Eqs.(2c),(2d),(2e) and (2f) are non-convex. These characteristics (free final time, non-convex constraints) make it difficult to solve the original problem in its current form.

III. PROPOSED ALGORITHM

To obtain an efficient sub-optimal solution to the problem, we apply a sequential convex programming (SCP) method. This method decomposes the original non-convex trajectory optimization problem into a sequence of convex sub-problems, enabling real-time computation. The main non-convex problem is approximated as a convex subproblem formulation, and the subproblem is iteratively solved until the solution converges

A. Convex Subproblem

The original problem in Eqs.(1)-(2) can be approximated as the following sub-problem. For each l^{th} iteration, the proposed algorithm solves the following convex sub-problem.

$$\min_{\mathbf{p}^{(l)},\mathbf{v}^{(l)},\mathbf{a}^{(l)},\sigma^{(l)}} J = \sigma^{(l)} + w \sum_{i,k} ||\mathbf{a}_i^{(l)}[k]|| \tag{3}$$

subject to

$$\mathbf{p}_i^{(l)}[0] = \mathbf{p}_i^0;\ \ \mathbf{p}_i^{(l)}[T] = \mathbf{p}_i^f \tag{4a}$$

$$\mathbf{v}_i^{(l)}[0] = \mathbf{v}_i^{(l)}[T] = [V, 0, 0]^T \tag{4b}$$

$$\begin{aligned}\mathbf{p}_i^{(l)}[k+1] = \mathbf{p}_i^{(l)}[k] + \mathbf{v}_i^{(l)}[k]\sigma^{(l-1)}\Delta\tau + \mathbf{v}_i^{(l-1)}[k]\sigma^{(l)}\Delta\tau\\
-\mathbf{v}_i^{(l-1)}[k]\sigma^{(l-1)}\Delta\tau\end{aligned} \tag{4c}$$

$$\begin{aligned}\mathbf{v}_i^{(l)}[k+1] = \mathbf{v}_i^{(l)}[k] + \mathbf{a}_i^{(l)}[k]\sigma^{(l-1)}\Delta\tau + \mathbf{a}_i^{(l-1)}[k]\sigma^{(l)}\Delta\tau\\
-\mathbf{a}_i^{(l-1)}[k]\sigma^{(l-1)}\Delta\tau\end{aligned} \tag{4d}$$

$$||\mathbf{v}_i^{(l)}[k]|| \leq v_{max};\ \ ||\mathbf{a}_i^{(l)}[k]|| \leq a_{max} \tag{4e}$$

$$\begin{aligned}D^2 \leq 2(\mathbf{p}_i^{(l-1)}[k] - \mathbf{p}_j^{(l-1)}[k])^T(\mathbf{p}_i^{(l)}[k] - \mathbf{p}_j^{(l)}[k])\\
-||\mathbf{p}_i^{(l-1)}[k] - \mathbf{p}_j^{(l-1)}[k]||^2\end{aligned} \tag{4f}$$

$$\begin{aligned}D_{INT}^2 \leq 2(\mathbf{p}_i^{(l-1)}[k] - \mathbf{p}_{INT}[k])^T(\mathbf{p}_i^{(l)}[k] - \mathbf{p}_{INT}[k])\\
-||\mathbf{p}_i^{(l-1)}[k] - \mathbf{p}_{INT}[k]||^2\end{aligned} \tag{4g}$$

First, Eq.(1) in the main problem is converted to Eq.(3), and Eqs.(2c)-(2d) are approximated as Eqs.(4c)-(4d). This approximation is mainly motivated by [4]. We adopt an auxiliary variable σ, fixed maximum time step T, and normalized fixed time interval $\Delta\tau \equiv 1/T$. Then the varying final time t_f in Eq.(1) is replaced as σ, and the time step Δt in Eqs.(2c)-(2d) is replaced as $\sigma\Delta\tau$. Then, a first-order Taylor series approximation is applied to Eqs.(2c)-(2d) with respect to the variable σ, $\mathbf{p}_i[k]$ and $\mathbf{v}_i[k]$. Second, the non-convex collision avoidance constraints Eqs.(2f)-(2g) in the main problem are approximated as Eqs.(4f)-(4g) by following the convexification techniques used in [5].

Algorithm 1 Sequential Convex Programming

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Require: Initial guess $\{\mathbf{p}^{(0)}, \mathbf{v}^{(0)}, \mathbf{a}^{(0)}, \sigma^{(0)}\}$, tolerance $\epsilon$
Set $l \leftarrow 1$
while $\|J^{(l)} - J^{(l-1)}\| > \epsilon$ do
    Solve the convex subproblem in Eq.(3)-(4)
    Obtain the solution $\{\mathbf{p}^{(l)}, \mathbf{v}^{(l)}, \mathbf{a}^{(l)}, \sigma^{(l)}\}$
    Set $l \leftarrow l + 1$
end while
```

B. Avoidance and Separation Strategy

Although the eVTOL is capable of 3-dimensional mobility, the severe 3D maneuver is not recommended when passengers are onboard. Depending on the threat level of the intruder, it is one of the important design characteristics to determine what kind of acceleration dimension would be used, to maximize the comfort feeling of onboard passengers. In the proposed problem formulation, the eVTOL trajectory is determined by the control history of acceleration $\mathbf{a}_i[k]$. The acceleration is a 3D vector with a_x component (along with corridor - forward acceleration/backward deceleration), a_y (lateral - left,right) and a_z (altitude up/down) component.

1) controlling a_x only: It is the simplest form of avoidance control. By only controlling x-component of the acceleration, each eVTOL can accelerate or decelerate only along with the forward/backward direction. This movement strategy is similar to a train or a subway - it can only change its forward speed and it cannot change its moving direction. As there is no lateral and vertical acceleration, additional constraints are required in the problem formulation ($a_y = 0$, $a_z = 0$). Then, each eVTOL still follows the given reference trajectory in the corridor, only changing its forward speed.

2) controlling a_x and a_y: This is a two-dimensional strategy for collision avoidance and separation. By controlling both a_x and a_y components of the acceleration, each eVTOL can also change its moving direction while keeping the same flight altitude. This movement is similar to a ground vehicle or a car on flat terrain. As there is no vertical acceleration, additional constraints are required in the problem formulation ($a_z = 0$).

3) controlling all components a_x,a_y and a_z: The third strategy is to control all acceleration components at the same time. In this way, each eVTOL can fully exploit its 3D mobility characteristics including its vertical movement (altitude change). By controlling all components, an efficient flight trajectory can be obtained to minimize flight time and overall control efforts to avoid the intruder. However, as the eVTOL has passengers, severe 3D mobility may provide an uncomfortable feeling to the passengers. Careful design for the three-dimensional acceleration would be required.

IV. NUMERICAL RESULTS

We considered an AAM scenario with five eVTOLs ($N = 5$) that fly along with the given straight line-shaped AAM corridor from $(0m, 0m, 500m)$ to $(2000m, 0m, 500m)$. Each eVTOL has flight speed $V = 50m/s$ and maximum flight speed $V_{max} = 100m/s$. eVTOLs need to keep a minimum mutual separation distance of $D = 100m$. Also, all eVTOLs should keep safety distance $D_{INT} = 150m$ against the intruder to avoid any potential risk of collision. For discrete-time optimization, the maximum number of time steps is set as $T = 100$. Fig.1 shows the optimization result with the stationary pop-up intruder located at $(800m, 25m, 500m)$. For ease of analysis, the second avoidance and separation strategy (controlling a_x and a_y) is applied by adding an additional constraint of $a_z = 0$ during optimization. The sequential convex programming iteration stopped at its 13th iteration.

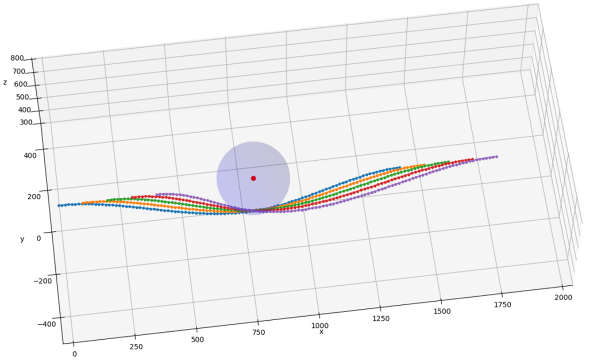

Fig. 1. Collision-free eVTOL trajectories (N=5)

V. CONCLUSION

This study suggested an iterative convex optimization framework for multi-eVTOL collision-free trajectory planning under the risk of a sudden unidentified aerial intruder. Further studies will include various case studies with realistic operation scenario parameters, more eVTOLs, more dynamic intruder modeling, CNSi modeling, and risk modeling.

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