Multi-User Beamforming under Per-Antenna Power Constraint

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***Abstract*—This paper proposes a design of multi-user beamformer under a per-antenna power constraint (PAPC). The proposed beamforming method effectively utilizes the antenna power and maximizes the beamforming gain while suppressing the inter-user interference. The resulting high beamforming gain and low interference can lead to the high sum-rate. All the processes of proposed beamforming method demand only linear operations, and this guarantees the low computational complexity. The simulation results show that the proposed method can provide the high sum-rate with significantly less computational complexity compared to existing beamforming methods that use complex optimization solvers to satisfy the PAPC.**

***Index Terms*—Per-antenna power constraint, multi-user beamforming, computational complexity.**

I. Introduction

In a multi-user wireless communication system, the deployment of multiple antennas at the base station (BS) can increase the sum-rate [1], [2]. The maximum sum-rate can be achieved with the dirty paper coding (DPC), but its computational complexity is unacceptably high to apply in practice [3]. As an appealing substitute, linear beamformers were developed that have significantly low complexity with sub-optimal performance instead [4], [5]. A zero-forcing (ZF) beamformer, especially, provides fine performance with sufficiently low complexity [6]–[8].

Under the per-antenna power constraint (PAPC), however, the design of ZF beamformer becomes non-trivial and loses its benefit of decent performance and low complexity [9], [10]. In practice, the BS has individual power amplifier for each antenna, and the transmit power is limited by the maximum power assigned to each antenna [9], [11]. The PAPC, therefore, has to be considered for practical beamformer designs.

Most of beamforming methods were designed by organizing optimization problems with the PAPC as a constraint. By considering a relaxed PAPC, a beamformer in [12] was designed to maximize a weighted sum-rate. In [13], a beamformer was designed to balance the weighted data rates of users with the PAPC. The beamformer design in [14] was proposed to update normalized beamformers and power distribution over the normalized beamformers alternately. In [15], a relaxed convex optimization problem was solved to maximize the sum-rate with the zero interference constraint and PAPC. For a spatially was designed by formulating a signal-to-interference-plus-noise ratio (SINR) balancing problem. In [17], a beamformer design was proposed to maximize the minimum data rate by solving a regularized dual problem. In [18], a transmit power minimization problem was formulated under the PAPC, and its solution was obtained by using the duality of primal- and dual-optimization problems. While the PAPC has been a long-standing problem, and many previous works, including [12]-[18], have tackled the PAPC, they barely achieved both fine performance and low complexity, which were possible under the total transmit power constraint.

In this paper, a low complexity linear beamformer design is proposed that can effectively satisfy the PAPC with the high sum-rate where all the processes can be conducted only with linear operations. Although the proposed beamformer design has an iterative structure, the finite number of iterations is guaranteed independent from the design parameters, while most of iterative methods, e.g., [13], [14], [19], may not be able to ensure their convergence without properly setting system parameters. The simulation results verify that the proposed beamforming method can achieve the high sum-rate, comparable to the methods using complicated optimization solvers [13], [15], with significantly low complexity.

In the rest of paper, system and channel models are described in Section II. The details of proposed beamforming method is explained in Section III. In Section IV, the performance of proposed beamforming method is compared with the existing methods in the metric of complexity and sum-rate. The conclusion, then, is presented in Section V.

Notations: A bold face capital and small letter denote a matrix and a vector. The transpose, Hermitian transpose, element-wise conjugate, ℓ-2 norm, and Frobenius norm are represented as $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$, $(\cdot)^{*}$, $\|\cdot\|_2$, and $\|\cdot\|_{\mathrm{F}}$. For a matrix $\mathbf{A}$, its pseudo-inverse, a-th column, and b-th row are remarked as $\mathbf{A}^{\dagger}$, $(\mathbf{A})_{(:,a)}$, and $(\mathbf{A})_{(b,:)}$. The b-th component of vector $\mathbf{a}$ is written as $(\mathbf{a})_{(b)}$. The $a \times a$ identity matrix and $a \times 1$ all zero vector are denoted as $\mathbf{I}_a$ and $\mathbf{0}_a$, and $\mathrm{diag}(\mathbf{a})$ is a diagonal matrix where the diagonal elements are the components of $\mathbf{a}$. The set of complex numbers, set of real numbers, and set of non-negative real numbers are remarked as $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{R}_+$. The Hadamard product is represented as $\odot$. The real part, phase, and absolute value of a complex number a are denoted as $\text{Real}\{a\}$, $\angle a$, and $|a|$. The function $\text{idx}[\mathcal{I}\{i\}]$, $i \in \mathcal{I}$ outputs the index of element i in a set $\mathcal{I}$, and the cardinality of the set is written as $\mathcal{C}(\mathcal{I})$.

II. System and channel models

A single cell multi-user multiple-input single-output (MU-MISO) system is considered with M BS antennas and K single antenna users. The received signal at the k-th user is

$$y_k = \mathbf{h}_k^{\mathrm{H}} \mathbf{f}_k s_k + \sum_{\substack{\ell=1 \\ \ell \neq k}}^{K} \mathbf{h}_k^{\mathrm{H}} \mathbf{f}_\ell s_\ell + n_k, \tag{1}$$

where $\mathbf{h}_k \in \mathbb{C}^{M\times 1}$ is the channel from the BS to the k-th user, $\mathbf{f}_k \in \mathbb{C}^{M\times 1}$ is the BS beamformer to support the k-th user, s_k is the transmit symbol for the k-th user, and $n_k \sim \mathcal{CN}(0, \sigma^2)$ is the additive white Gaussian noise with the variance σ^2. The transmit symbol s_k has mean $\mathbb{E}[s_k] = 0$ and variance $\mathbb{E}[|s_k|^2] = 1$ where the symbol for each user is statistically independent.

To design the beamformers $\mathbf{f}_1, \cdots, \mathbf{f}_K$, practical power constraints are considered as

$$\sum_{k=1}^{K} \|\mathbf{f}_k\|_2^2 \leq P_{\text{tot}}, \tag{2}$$

$$\max_{m\in\{1,\cdots,M\}} \sum_{k=1}^{K} \left|(\mathbf{f}_k)_{(m)}\right|^2 \leq P_{\text{ant}}, \tag{3}$$

where (2) constrains the total transmit power of beamformer by the maximum total transmit power P_{tot}, and (3) limits the per-antenna power of beamformer by the maximum antenna transmit power P_{ant}. The beamformer needs to satisfy the two power constraints simultaneously. In this paper, we set the maximum antenna power to be $P_{\text{ant}} = P_{\text{tot}}/M$, and this makes the PAPC in (3) the sufficient condition for the total power constraint in (2).

The channel from the BS to each user is modeled as [20]

$$\mathbf{h}_k = \mathbf{R}_k^{\frac{1}{2}} \mathbf{g}_k, \tag{4}$$

where $\mathbf{R}_k = \mathbb{E}\left[\mathbf{h}_k \mathbf{h}_k^{\mathrm{H}}\right]$ is the spatial correlation of channel vector, and the random vector $\mathbf{g}_k$ follows complex normal distribution $\mathcal{CN}(\mathbf{0}_M, \mathbf{I}_M)$. The entire channel matrix is the concatenation of each channel vectors $\mathbf{H} = [\mathbf{h}_1, \cdots, \mathbf{h}_K]$. The spatial correlation $\mathbf{R}_k$ is constructed with the exponential model as

$$\mathbf{R}_k = \begin{bmatrix} 1 & r_k & \cdots & r_k^{(M-1)} \\ r_k^* & 1 & \cdots & r_k^{(M-2)} \\ \vdots & \vdots & \ddots & \vdots \\ (r_k^*)^{(M-1)} & (r_k^*)^{(M-2)} & \cdots & 1 \end{bmatrix}, \tag{5}$$

where $r_k \in \mathbb{C}$ has magnitude $|r_k| < 1$ and phase $0 \leq \angle r_k < 2\pi$ [21], [22].

The design objective of the proposed beamformer is the maximization of sum-rate where the optimization problem can be defined as

$$\begin{aligned} \underset{\mathbf{f}_1,\cdots,\mathbf{f}_K}{\text{maximize}} \quad & \sum_{k=1}^{K} \log_2 \left(1 + \frac{\left|\mathbf{h}_k^{\mathrm{H}} \mathbf{f}_k\right|^2}{\sigma^2 + \sum_{\substack{\ell=1 \\ \ell\neq k}}^{K} \left|\mathbf{h}_k^{\mathrm{H}} \mathbf{f}_\ell\right|^2} \right) \\ \text{subject to} \quad & \max_{m\in\{1,\cdots,M\}} \sum_{k=1}^{K} \left|(\mathbf{f}_k)_{(m)}\right|^2 \leq P_{\text{ant}}. \end{aligned} \tag{6}$$

Both the objective and constraint functions in the problem (6), however, are non-convex, which makes the problem challenging. To tackle the problem (6), we propose a beamformer design that provides a suboptimal solution but with low computational complexity.

III. Proposed beamformer design

A simple approach to produce a beamformer satisfying the PAPC in (3) is downscaling a beamformer that is designed under the total transmit power constraint in (2) only. The scaled-down beamformer, however, can suffer from beamforming gain degradation. For example, an ordinary ZF beamformer would be scaled-down to satisfy the PAPC as

$$\mathbf{F}_{\text{nZF}} = [\mathbf{f}_{\text{nZF},1}, \cdots, \mathbf{f}_{\text{nZF},K}], \tag{7}$$

$$\mathbf{f}_{\text{nZF},k} = \nu_{\text{PAPC}} \mathbf{f}_{\text{ZF},k}, \quad k \in \{1, \cdots, K\}, \tag{8}$$

$$\mathbf{f}_{\text{ZF},k} = \nu_{\text{tot}} \frac{\left(\left(\mathbf{H}^{\mathrm{H}}\right)^{\dagger}\right)_{(:,k)}}{\left\|\left(\left(\mathbf{H}^{\mathrm{H}}\right)^{\dagger}\right)_{(:,k)}\right\|_2}, \quad k \in \{1, \cdots, K\}, \tag{9}$$

where ν_{PAPC} and ν_{tot} are the scaling terms to satisfy the PAPC and the total transmit power constraint, respectively, which are calculated as

$$\nu_{\text{PAPC}} = \sqrt{\frac{P_{\text{ant}}}{\sum_{k=1}^{K} \left|(\mathbf{f}_{\text{ZF},k})_{(m_{\max})}\right|^2}}, \tag{10}$$

$$\nu_{\text{tot}} = \sqrt{\frac{P_{\text{tot}}}{K}}. \tag{11}$$

In (10), $m_{\max}$ is the index of antenna that uses the highest antenna power $m_{\max} = \text{argmax}_{m\in\{1,\cdots,M\}} \sum_{k=1}^{K} |(\mathbf{f}_{\text{ZF},k})_{(m)}|^2$. The transmit power of the normalized and unnormalized ZF beamformers can be compared as

$$\begin{aligned} \sum_{k=1}^{K} \|\mathbf{f}_{\text{nZF},k}\|_2^2 &= \sum_{k=1}^{K} \frac{P_{\text{ant}} \|\mathbf{f}_{\text{ZF},k}\|_2^2}{\sum_{\ell=1}^{K} \left|(\mathbf{f}_{\text{ZF},\ell})_{(m_{\max})}\right|^2} \\ &\overset{(a)}{\leq} \sum_{k=1}^{K} \frac{P_{\text{tot}} \|\mathbf{f}_{\text{ZF},k}\|_2^2}{M\left(\frac{1}{M}\sum_{m=1}^{M}\sum_{\ell=1}^{K} \left|(\mathbf{f}_{\text{ZF},\ell})_{(m)}\right|^2\right)} \\ &= P_{\text{tot}} \\ &= \sum_{k=1}^{K} \|\mathbf{f}_{\text{ZF},k}\|_2^2, \end{aligned} \tag{12}$$

where the equality of (a) holds when all the antenna power of unnormalized ZF beamformer is the same, which is extremely

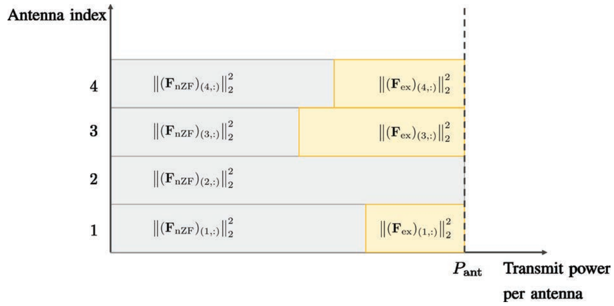


Fig. 1: Transmit power per-antenna with $M = 4$.

unlikely. The normalized beamformer, hence, usually exploits much lower transmit power than the unnormalized beamformer, and a large amount of antenna power is left unused.

As one possible approach to utilize the unused antenna power, we propose to add an extra beamformer $\mathbf{F}_{\text{ex}}$ to the normalized beamformer $\mathbf{F}_{\text{nZF}}$[1]

$$\mathbf{F} = \mathbf{F}_{\text{nZF}} + \mathbf{F}_{\text{ex}}. \tag{13}$$

With a proper design of the extra beamformer $\mathbf{F}_{\text{ex}}$, the beamforming gain of combined beamformer $\mathbf{F}$ can increase. A conceptual example for understanding is depicted in Fig. 1. The normalized ZF beamformer $\mathbf{F}_{\text{nZF}}$ is downscaled to satisfy the PAPC of all antennas, and only the second antenna can utilize its maximum antenna power P_{ant}. If we would design an extra beamformer $\mathbf{F}_{\text{ex}}$ for the first, third, and fourth antennas by exploiting their unused power $P_{\text{ant}} - \left\|(\mathbf{F}_{\text{nZF}})_{(m,:)}\right\|_2^2$, $m \in \{1, 3, 4\}$, the combined beamformer $\mathbf{F}$ in (13) can provide an improved beamforming gain with full use of the transmit power.

The simple increase of the beamforming gain for a specific user, however, may lead to additional interference to other users. Therefore, we design the extra beamformer as

$$\mathbf{F}_{\text{ex}} = \mathbf{W}\mathbf{A}, \tag{14}$$

where $\mathbf{W}$ is an interference managing beamformer, and $\mathbf{A} = \text{diag}(\mathbf{a})$ is a diagonal matrix determining the power distribution over users with diagonal elements $(\mathbf{a})_{(k)}$. By combining multiple effective extra beamformers, the initial scaled-down beamformer can gradually improved. For instance, the columns of $\mathbf{W}$ can be normalized ZF beamformer. The overall process of proposed beamformer design is in Algorithm 1.

In each algorithm iteration, the index set $\mathcal{I}_n$ consists of the antennas that use power less than P_{ant}, in other words, the antennas with available power not in use. The extra beamformer $\widehat{\mathbf{W}}_n$, then, is designed as a ZF beamformer for the antennas in $\mathcal{I}_n$. By having the initial and extra beamformers as the ZF beamformer, all updated beamformers $\mathbf{F}_n$ become the ZF beamformers, enforcing the zero inter-user interference. Before the extra beamformer $\widehat{\mathbf{W}}_n$ is combined with the previous beamformer $\mathbf{F}_{n-1}$, the dimension of $\widehat{\mathbf{W}}_n$ is adjusted as $\mathbf{W}_n$,

[1]The full derivation of proposed beamforming technique can be found in [23]

Algorithm 1 Proposed low complexity beamformer design under the PAPC

Step 1: Set initial beamformer $\mathbf{F}_0 = \mathbf{F}_{\text{nZF}}$ in (7) and parameter $0 < \epsilon < 1$
Step 2: **For** $1 \le n \le M - K$
Step 3: Organize antenna index set:

$$\mathcal{I}_n = \left\{ m : \sum_{k=1}^{K} \left|(\mathbf{F}_n)_{(m,k)}\right|^2 < P_{\text{ant}} \right\}$$

Step 4: $\mathcal{C}(\mathcal{I}_n) < K$: **break**
Step 5: **End if**
Step 6: Calculate an extra beamformer for the antenna set:

$$\widehat{\mathbf{W}}_n = \left(\widehat{\mathbf{H}}_n^{\text{H}}\right)^{\dagger} \in \mathbb{C}^{\mathcal{C}(\mathcal{I}_n) \times K}$$
$$\widehat{\mathbf{H}}_n = (\mathbf{H})_{(\mathcal{I}_n,:)}$$

Step 7: Adjust beamformer dimension:

$$(\mathbf{W}_n)_{(m,:)} = \begin{cases} (\widehat{\mathbf{W}}_n)_{(\text{idx}[\mathcal{I}_n\{m\}],:)}, & m \in \mathcal{I}_n \\ \mathbf{0}_K^{\text{T}}, & m \notin \mathcal{I}_n \end{cases}$$

Step 8: Set coefficient matrix with (15): $\mathbf{A}_n = \text{diag}(\mathbf{a}_n)$
Step 9: Combine beamformers: $\mathbf{F}_n = \mathbf{F}_{n-1} + \mathbf{W}_n\mathbf{A}_n$
Step 10: **If** $\sum_{k=1}^{K} \left\|(\mathbf{F}_n)_{(:,k)}\right\|_2^2 > (1-\epsilon)P_{\text{tot}}$: **break**
Step 11: **End if**
Step 12: **End for**
Step 13: Output beamformer: $\mathbf{F}_{\text{out}} = \mathbf{F}_n$

and its power allocation is determined by $\mathbf{A}_n$. The coefficient matrix $\mathbf{A}_n = \text{diag}(\mathbf{a}_n)$ is designed as

$$(\mathbf{a}_n)_{(k)} = \frac{\alpha_n}{\left\|(\mathbf{W}_n)_{(:,k)}\right\|_2}, \quad k \in \{1, \cdots, K\}, \tag{15}$$

where α_n is the coefficient to satisfy the PAPC.

Based on the structure of beamformer combination and coefficient matrix, the sum-rate maximization problem can be formulated with respect to α_n as

$$\underset{\hat{\alpha}_n \in \mathbb{C}}{\text{argmax}} \sum_{k=1}^{K} \log_2 \left(1 + \frac{1}{\sigma^2} \left|\mathbf{h}_k^{\text{H}} \left(\mathbf{F}_{n-1} + \mathbf{W}_n\mathbf{A}_n\right)_{(:,k)}\right|^2\right) \tag{16-a}$$

$$\text{subject to } \sum_{k=1}^{K} \left|(\mathbf{F}_{n-1} + \mathbf{W}_n\mathbf{A}_n)_{(m,k)}\right|^2 \le P_{\text{ant}}, \quad m \in \{1, \cdots, M\}, \tag{16-b}$$

$$\angle\left(\mathbf{h}_k^{\text{H}}(\mathbf{F}_{n-1})_{(:,k)}\right) = \angle\left(\mathbf{h}_k^{\text{H}}(\mathbf{W}_n\mathbf{A}_n)_{(:,k)}\right), \quad k \in \{1, \cdots, K\}, \tag{16-c}$$

$$\mathbf{A}_n = \text{diag}(\mathbf{a}_n), \tag{16-d}$$

$$(\mathbf{a}_n)_{(k)} = \frac{\hat{\alpha}_n}{\left\|(\mathbf{W}_n)_{(:,k)}\right\|_2}, k \in \{1, \cdots, K\}, \tag{16-e}$$

where (16-b) is the PAPC, and (16-c) aligns the two vector products $\mathbf{h}_k^{\text{H}}(\mathbf{F}_{n-1})_{(:,k)}$ and $\mathbf{h}_k^{\text{H}}(\mathbf{W}_n\mathbf{A}_n)_{(:,k)}$.

In the objective function in (16-a), we can first consider the maximization of k-th user data rate

$$\underset{\hat{\alpha}_n \in \mathbb{C}}{\operatorname{argmax}} \ \log_2\left(1+\frac{1}{\sigma^2}\left|\mathbf{h}_k^{\mathrm{H}}\left(\mathbf{F}_{n-1}+\mathbf{W}_n\mathbf{A}_n\right)_{(:,k)}\right|^2\right), \quad (17)$$

which can be reformulated as

$$\begin{aligned}\underset{\hat{\alpha}_n \in \mathbb{C}}{\operatorname{argmax}} & \left|\mathbf{h}_k^{\mathrm{H}}\left(\mathbf{W}_n\mathbf{A}_n\right)_{(:,k)}\right|^2 \\ & +2\cdot\operatorname{Real}\left\{\mathbf{h}_k^{\mathrm{H}}\left(\mathbf{F}_{n-1}\right)_{(:,k)}\left(\mathbf{h}_k^{\mathrm{H}}\left(\mathbf{W}_n\mathbf{A}_n\right)_{(:,k)}\right)^*\right\}. \quad (18)\end{aligned}$$

The ZF beamformer structure of $(\mathbf{F}_{n-1})_{(:,k)}$ and $(\mathbf{W}_n)_{(:,k)}$ gives positive products $\mathbf{h}_k^{\mathrm{H}}(\mathbf{F}_{n-1})_{(:,k)} \in \mathbb{R}_+$ and $\mathbf{h}_k^{\mathrm{H}}(\mathbf{W}_n\mathbf{A}_n)_{(:,k)} \in \mathbb{R}_+$, and the coefficient $\hat{\alpha}_n$ is also restricted to be a real value by the alignment constraint (16-c). With the constraints (16-c)-(16-e), the optimization in (18) becomes

$$\begin{aligned}\underset{\hat{\alpha}_n \in \mathbb{R}}{\operatorname{argmax}} & \ \frac{\hat{\alpha}_n^2}{\|(\mathbf{W}_n)_{(:,k)}\|_2^2}\left|\mathbf{h}_k^{\mathrm{H}}\left(\mathbf{W}_n\right)_{(:,k)}\right|^2 \\ & +\frac{2\hat{\alpha}_n}{\|(\mathbf{W}_n)_{(:,k)}\|_2}\mathbf{h}_k^{\mathrm{H}}\left(\mathbf{F}_{n-1}\right)_{(:,k)}\left(\mathbf{h}_k^{\mathrm{H}}\left(\mathbf{W}_n\right)_{(:,k)}\right)^* \\ = & \underset{\hat{\alpha}_n \in \mathbb{R}}{\operatorname{argmax}} \ \hat{\alpha}_n. \quad (19)\end{aligned}$$

The last simplified objective function in (19) implies that the increase of single coefficient $\hat{\alpha}_n$ can maximize the data rate of k-th user. In addition, the independence of last objective function in (19) from the user index k means that the optimization of $\hat{\alpha}_n$ leads to the data rate maximization of all users. We, hence, can find a solution for the sum-rate maximization problem (16-a) by obtaining the maximum feasible $\hat{\alpha}_n$.

As the constraints (16-c)-(16-e) are considered in (19), the feasible set of $\hat{\alpha}_n$ is defined by the PAPC in (16-b). The feasible set that is determined by the m-th antenna PAPC is

$$\sum_{k=1}^{K}\left|(\mathbf{F}_{n-1})_{(m,k)}+\frac{\hat{\alpha}_n}{\left\|(\mathbf{W}_n)_{(:,k)}\right\|_2}(\mathbf{W}_n)_{(m,k)}\right|^2 \leq P_{\mathrm{ant}}, \quad (20)$$

which can be rewritten as

$$\begin{aligned}&\hat{\alpha}_n^2\left\|(\mathbf{W}_n)_{(m,:)}\odot\overline{\mathbf{w}}_n\right\|_2^2 \\ &+2\cdot\operatorname{Real}\left\{\hat{\alpha}_n(\mathbf{F}_{n-1})_{(m,:)}^*\left((\mathbf{W}_n)_{(m,:)}\odot\overline{\mathbf{w}}_n\right)^{\mathrm{T}}\right\} \\ &+\left\|(\mathbf{F}_{n-1})_{(m,:)}\right\|_2^2-P_{\mathrm{ant}}\leq 0, \quad (21)\end{aligned}$$

where $\overline{\mathbf{w}}_n = \left[\left\|(\mathbf{W}_n)_{(:,1)}\right\|_2^{-1},\cdots,\left\|(\mathbf{W}_n)_{(:,K)}\right\|_2^{-1}\right]$. In the feasible set of $\hat{\alpha}_n$ in (21), the maximum value for the m-th antenna can be computed as

$$\begin{aligned}\hat{\alpha}_{n,m} = & \frac{1}{\left\|(\mathbf{W}_n)_{(m,:)}\odot\overline{\mathbf{w}}_n\right\|_2^2} \\ & \times\Bigg(-\operatorname{Real}\left\{(\mathbf{F}_{n-1})_{(m,:)}\left((\mathbf{W}_n)_{(m,:)}\odot\overline{\mathbf{w}}_n\right)^{\mathrm{H}}\right\} \\ & +\Bigg(\left|\operatorname{Real}\left\{(\mathbf{F}_{n-1})_{(m,:)}\left((\mathbf{W}_n)_{(m,:)}\odot\overline{\mathbf{w}}_n\right)^{\mathrm{H}}\right\}\right|^2 \\ & -\left\|(\mathbf{W}_n)_{(m,:)}\odot\overline{\mathbf{w}}_n\right\|_2^2\left(\left\|(\mathbf{F}_{n-1})_{(m,:)}\right\|_2^2-P_{\mathrm{ant}}\right)\Bigg)^{1/2}\Bigg). \quad (22)\end{aligned}$$

Satisfying all the M PAPC, the maximum $\hat{\alpha}_n$, i.e., the solution of the problem (16-a), is obtained as

$$\alpha_n = \underset{\hat{\alpha}_n \in \{\hat{\alpha}_{n,1},\cdots,\hat{\alpha}_{n,M}\}}{\operatorname{argmin}} \hat{\alpha}_n. \quad (23)$$

In each algorithm iteration, the design of $\mathbf{A}_n$ with α_n in (23) lets an antenna in the set $\mathcal{I}_n$ use its maximum power P_{ant}, and the number of antennas utilizing their maximum antenna power increases over algorithm iterations. Under the maximum algorithm iteration $M-K$ in Step 2, the proposed algorithm includes two stopping criteria. One in Step 4 is about the the cardinality of antenna index set. To compute the ZF beamformer for the n-th antenna index set $\mathcal{I}_n$, its cardinality needs to be larger than or equal to the number of users $\mathcal{C}(\mathcal{I}_n) \geq K$. The other stopping criterion in Step 10 depends on the total transmit power. When the n-th updated beamformer $\mathbf{F}_n$ exploits total transmit power more than $(1-\epsilon)P_{\mathrm{tot}}$, the proposed algorithm stops. With sufficiently small ϵ, the proposed beamformer design can exploit most of total transmit power, and the proposed beamformer design ensures high data rates for users.

TABLE I: Comparison of computational complexities with $M > K$ and optimization accuracy ϵ_{acc}.

Beamforming method	Complexity
Proposed	$\mathcal{O}\left(M^2K^2\right)$
[13]	$\mathcal{O}\left(L_{\mathrm{algo}}M^3K^3\sqrt{M+K}\log 1/\epsilon_{\mathrm{acc}}\right)$
[15]	$\mathcal{O}\left(M^{3.5}K^{3.5}\log 1/\epsilon_{\mathrm{acc}}\right)$

IV. Complexity analysis and simulation results

In this section, the proposed low complexity beamformer design is compared with other beamforming methods in [13] and [15] that use optimization solvers to obtain beamformers. The two beamformers in [13] and [15] are designed under the PAPC, where the balancing problem of weighted data rates is solved in [13], and the sum-rate maximization problem with zero interference constraints is considered in [15].

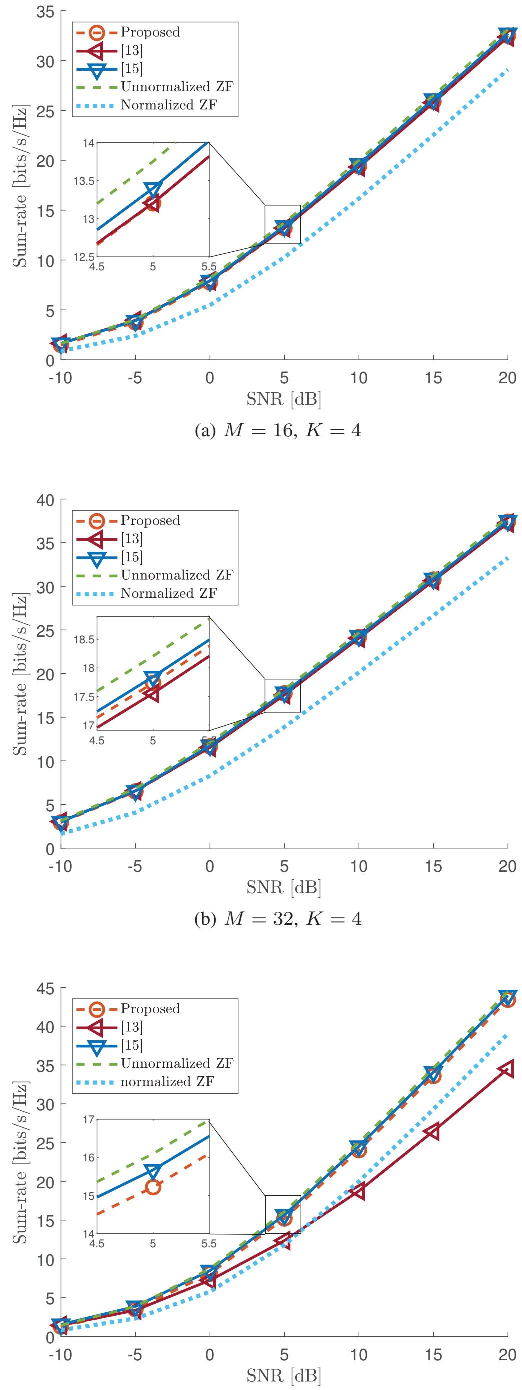
A. Complexity comparison

The computational complexities of the proposed and other beamformer designs in [13] and [15] are compared in Table I when the number of BS antennas M is larger than the number of users K. In terms of both M and K, it is clearly shown that the complexity of proposed beamformer design is remarkably lower than those of other beamformer designs in [13] and [15]. Without any closed form solution, the two beamformer designs in [13] and [15] require the use of optimization solvers, and their complexities additionally increase with the accuracy of optimization solver ϵ_{acc}. In the complexity of beamformer in [13], the unbounded number of algorithm iteration L_{algo} also can cause further increase of complexity. Although the proposed beamformer design also operates iteratively, in addition to the finite iteration number, the closed form solution is derived for each iteration, which requires only linear operations, making the proposed beamformer design highly practical.

B. Data rate comparison

For the comparison of sum-rates of beamformers, we consider M BS antennas and K users. For the spatial correlation of channel vectors, the angle $\angle r_k$ is distributed uniformly in $[0, 2\pi)$ with the magnitude $|r_k| = 0.6$. The transmit signal-to-noise ratio (SNR) is $\frac{P_{\text{tot}}}{\sigma^2}$, and the proposed beamformer design is operated with $\epsilon = 10^{-5}$. As additional references, two kinds of ZF beamformers, i.e., the unnormalized ZF beamformer $\mathbf{F}_{\text{ZF}} = [\mathbf{f}_{\text{ZF},1}, \cdots, \mathbf{f}_{\text{ZF},K}]$ that only satisfies the total transmit power constraint and the normalized ZF beamformer $\mathbf{F}_{\text{nZF}}$ that satisfies both the total and per-antenna power constraints, are also presented.

In Fig. 2, without the PAPC, the unnormalized ZF beamformer provides the highest sum-rate. The beamformer in [15], which maximizes the sum-rate using a complicated optimization solver, gives the second highest sum-rate. Under the PAPC, the proposed beamformer is designed with the notably low complexity, and its high sum-rate is close to those of the unnormalized ZF beamformer and the beamformer in [15]. The beamformer in [13] is another complicated beamformer and depicts a high sum-rate in Figs. 2a and 2b, but its sum-rate falls below that of the simply normalized ZF beamformer in Fig. 2c. Since the design purpose of beamformer in [13] is to balance the data rates of users, the sum-rate of beamformer in [13] grows slowly with SNR as the number of users increases.



(a) $M = 16$, $K = 4$

(b) $M = 32$, $K = 4$

(c) $M = 16$, $K = 6$

Fig. 2: Comparison of sum-rates over SNR with $M = 16$.

V. Conclusion

We proposed a beamformer design algorithm that effectively utilizes the transmit power under the PAPC. With the closed form solution for each algorithm iteration and finite iterations, the proposed beamformer design requires significantly low complexity compared to other beamformer designs that rely on the optimization solvers without any closed form solution. The simulation results, in addition, showed that that the proposed beamformer design can provide the high sum-rate that is close to the unnormalized ZF beamformer without the PAPC. Because of its high performance with low complexity, the proposed beamforming method is highly practical under the PAPR.

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