

# Stochastic Geometry Analysis of Position aided Beam Management

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## I. INTRODUCTION

Beamforming has gained significant attention due to its numerous benefits. Beamforming has increased transmission coverage, and also improved signal quality while decreasing its interference caused by omni-directional broadcasting. However beam impairment between transmitter and receiver occurs when mobility is introduced and this beam misalignment can severely degrade overall network performance [1]. In response, beam management plays a crucial role in maintaining network performance [2]. In this paper, we analyze the time that an effective beam is maintained in the LOS environment defined as beam coherence time. In addition, we express the ergodic throughput of the network through stochastic geometry analysis when error-prone position information is given to each mobile terminal, investigating the effect of beamwidth and velocity on the system.

## II. SYSTEM MODEL

Consider a scenario where position information is given to each Tx and Rx. Each position information has an error defined with an error radius. The actual position is located in given circle, with each error radius denoted as  $r_T, r_R$  for Tx and Rx respectively. The distance between Tx and Rx is denoted as  $D$  and a line of sight path between Tx and Rx is guaranteed. Tx and Rx sweep their beam candidates to find the optimal beam that has strongest received power [3]. When position information with error is given as in Fig. 1, sweeping angle is  $2\theta$  where

$$\theta = \arcsin\left(\frac{r_T + r_R}{D}\right). \quad (1)$$

We denote half beamwidth of Tx and Rx as  $\phi$ . Thus, the number of beam searched is  $(\frac{2\theta}{\phi})^2$ . The beam searching duration for single beam pair is denoted as  $\tau$ . In return, total beam searching overhead is  $\tau(\frac{2\theta}{\phi})^2$  when position information is given.

## III. BEAM COHERENCE TIME

In this section, the beam coherence time is defined and ergodic beam coherence time is analyzed. Due to mobility of Tx and Rx, the optimal beam pair may be misaligned. In this scenario, we say beam is a beam is misaligned when the mainlobe of each Tx and Rx does not point coherently at the other mainlobe. For example, if one of the mobile terminal's bore-sight no longer points within other terminal's mainlobe, beam is misaligned. The time duration from optimal beam pairing to beam misalignment is defined as beam coherence time denoted as  $T_c$ .

$$T_c = \frac{D}{v} \frac{\sin \phi}{\sin(\phi + \alpha)}, \quad (2)$$

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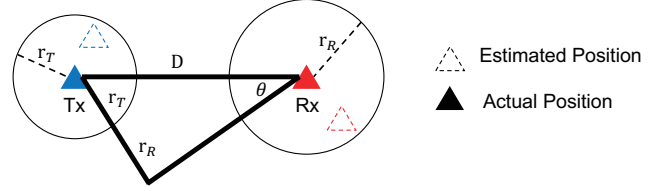


Fig. 1. Each Tx and Rx perform beam level sweeping with given position information.

where  $v$  is relative velocity of Rx and  $\alpha$  denotes relative moving direction of Rx. Assuming Rx moving towards Tx, relative moving direction is random variable which is uniformly distributed over  $(0, \pi/2)$ . Then ergodic beam coherence time is formulated as

$$\mathbb{E}[T_c] = \int_0^{\pi/2} \frac{v}{D} \frac{\pi}{2} \frac{\sin \phi}{\sin(\phi + \alpha)} d\alpha \quad (3)$$

$$= \frac{2D \sin \phi}{v\pi} \ln \frac{1 + \cos \phi}{\tan \phi(1 - \sin \phi)}. \quad (4)$$

The transmitter can send its data within beam coherence time excluding overhead caused by beam pairing. Then, ergodic throughput can be written as

$$\beta(\phi) = \left( \frac{2D \sin \phi}{v\pi} \ln \left( \frac{1 + \cos \phi}{\tan \phi(1 - \sin \phi)} \right) - \tau \left( \frac{2\theta}{\phi} \right)^2 \right) W \quad (5)$$

$$\times \log_2 \left( 1 + \frac{P_t \left( \frac{2\pi}{\phi} \right) \left( \frac{2\pi}{\phi} \right) D^{-\eta}}{N_0} \right), \quad (6)$$

where,  $W$  is bandwidth and  $\eta$  is pathloss exponent.

## IV. CONCLUDING REMARKS

In this paper, we have defined the beam coherence time and investigated it under the fast moving scenario. Through the ergodic beam coherence time, we have analyzed ergodic throughput with error-prone position aided scenario. The current work can be further expanded to optimize throughput by controlling numerous variable such as beamwidth and position acquisition time. Also, it is interesting to investigate such scenario under non line of sight environments.

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