MMSE-based Turbo Equalization Schemes with Damping for Coded OTFS Systems

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***Abstract*—This paper presents minimum mean squared error (MMSE)-based turbo equalization schemes with damping for coded orthogonal time frequency space (OTFS) systems in order to address a problematic underestimation in post-processed residual inter-data symbol interference (IDSI) when calculating a likelihood function for data symbol demapping with Gaussian approximation thereof. We suggest two alternatives in dampening: 1) *equalizer-fed* extrinsic log-likelihood ratio (LLR) and 2) *decoder-fed* extrinsic LLR. It is demonstrated that the two proposed damping methods have a better signal-to-noise ratio (SNR) vs. block error rate (BLER) performance than one without damping via the rigorous link-level performance evaluations over various channel scenarios.**

***Keywords*—turbo equalization, OTFS, MMSE, damping.**

I. INTRODUCTION

Framework and overall objectives of the future development of IMT for 2030 and beyond have been finally drafted in June 2023 [1], with being ahead of its approval by ITU-R Study Group 5 (SG 5) in September 2023 under Resolution ITU-R 1-8. This work is a sort of priming water to kick off the development of the Sixth-Generation (6G) mobile communications system deployed in the upcoming era. It is embracing six usage scenarios: 'Ubiquitous Connectivity,' 'Integrated AI and Communication,' 'Integrated Sensing and Communication,' 'Immersive Communication,' 'Massive Communication,' and 'Hyper Reliable & Low-Latency Communication.' The first three are newly introduced, and the others are an extension of 'Enhanced Mobile Broadband', 'Enhanced Machine-Type Communication', and 'Ultra Reliable & Low-Latency Communication' defined in IMT-2020 (so-called 5G), respectively. Furthermore, it is defining fifteen capabilities for IMT-2030 accomplished by six new capabilities and nine enhanced capabilities, for which a single or multiple values can be set for each usage scenario enumerated above. Regarding the capability of mobility, the research target is in the range of 500 to 1000 km/h, at which a defined QoS and seamless transfer between radio nodes can be achieved. Such a challenging mobility requirement may be necessitated for hyper-high-speed railway and low earth orbit (LEO) satellite communications.

Meanwhile, orthogonal time frequency space (OTFS) was devised as a Doppler-sturdy waveform via two-dimensional (2D) spreading from the delay-Doppler (DD) domain to the time-frequency (TF) domain on top of orthogonal frequency division multiplexing (OFDM) modulation for each multicarrier symbol [2]. By doing so, each data symbol undergoes all the channels spanned in the whole (or allocated) time-frequency resources, which can provide data-symbol-level diversity even in an uncoded system, but each data symbol is interfered with by some of the other data symbols due to entanglement in between them by delay and Doppler shift being inherent from time-varying multipath channels. The entanglement can be analyzed through an input-output relationship in the DD domain wherein the effective channel matrix can be viewed as a 2D (quasi-)circular convolution [3]. In order to fully exploit data-symbol-level diversity under the DD-domain effective channel with channel-inherent inter-data symbol interference (IDSI), therefore, a non-linear channel equalization scheme, taking IDSI into account to jointly detect or cancel out, such as maximum likelihood (ML), maximum *a posteriori* probability (MAP), and turbo equalizations should be employed [2-4]. In this paper, we consider a minimum mean squared error (MMSE)-based turbo equalization in a coded OTFS system. In the course of turbo iteration, the IDSI is canceled by the mean of data symbols (i.e., softly reconstructed data symbols) and its residual IDSI is suppressed by the MMSE filter where the level of projection onto the null space of the residual IDSI-associated channel being controlled by the variance of data symbols [4]. In every turbo iteration, the turbo-equalized data symbols need to be demapped to reckon the extrinsic log-likelihoods (LLRs) of coded bits comprising each data symbol, which are used to recover the original information/message bits by inputting to a channel decoder. When calculating the extrinsic LLRs, a likelihood function of the equalized data symbols is to be defined. It is difficult to assess the exact statistics of the post-processed residual IDSI, so Gaussian approximation of their distribution is borrowed as a simple but practical way in a real system [4]. However, the Gaussian approximation works well if the number of post-processed residual IDSI components is sufficiently large. Demapping the equalized data symbols with the Gaussian approximation may not guarantee the (near-)optimal performance if the number of channel paths, being equivalent to the number of the IDSI components, is not large enough to hold its tightness. In this paper, to cope with loss from the lack of accuracy in the Gaussian approximation, we propose two damping methods: 1) *equalizer-fed* extrinsic LLR damping and 2) *decoder-fed* extrinsic LLR damping. The extrinsic LLR from the equalization is dampened in the former method while *a priori* probability induced from an interleaved version of the extrinsic LLR from the channel decoder is dampened in the latter method. In a nutshell, such damping methods prevent the equalized- and decoder-fed extrinsic LLRs from being updated abruptly for each turbo iteration. The link-level performance evaluations are performed for the two proposed damping methods with respect to several candidate values of the damping factor in varied channel scenarios in comparison to the case without damping, to verify their efficacy and to observe the effect of the damping factor.

The remainder of the paper is organized as follows. A system model of the OTFS system under time-varying multipath channels is provided in Section II. Section III presents the MMSE-based turbo equalization scheme equipped with the proposed damping methods for OTFS, and the results of the link-level performance evaluations, in terms of SNR vs. BLER, are provided to validate the performance gain in Section IV. Finally, Section V draws the conclusion.

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Hereafter, $(\cdot)^T$, $(\cdot)^H$, $\mathrm{vec}(\cdot)$, and $\mathrm{vec}^{-1}_{M\times N}(\cdot)$ denote transpose, complex conjugate transpose (or Hermitian), vectorization, and de-vectorization into an $M\times N$ matrix, respectively. $a_{i,j}$ (or $[\mathbf{A}]_{i,j}$) and $\mathbf{a}_i$ (or $[\mathbf{A}]_i$) represent the *i*-th row and *j*-th column element and the *i*-th column of a matrix **A**, respectively. The expectation is denoted by $\mathrm{E}[\cdot]$. $\mathbf{I}_M$ and $\mathrm{diag}(\mathbf{a})$ signify the $M\times M$ identity matrix and the diagonal matrix whose diagonal entries are given as elements of **a**, respectively. $\mathbf{1}_{M\times N}$ and $\mathbf{0}_{M\times N}$ denote $M\times N$ all-zeros and all-ones matrices, respectively. $\otimes$ represents Kronecker product. It is assumed that column and row indices of a matrix are started from 0. $|\Xi|$ is a cardinality of a set Ξ. $\imath := \sqrt{-1}$ and $[K] := \{0,1,\ldots,K-1\}$ for $K(\in \mathbb{N})$.

II. System Model

In this section, we briefly review a system model for coded OTFS in baseband digital domain. Without loss of generality, a rectangular window after spreading (before despreading) is considered at the transmitter (receiver). Also, it is assumed to spread each data symbol in delay-Doppler (DD) domain into the whole resources in time-frequency (TF) domain when describing OTFS modulation/demodulation.

A. OTFS Modulation

A transport block comprised of information/message bits is channel-coded, possibly following cyclic redundancy check (CRC) generation and insertion. A sequence of coded bits fed from the channel encoder is modulated into a sequence of data symbols, called a codeword, typically by quadrature amplitude modulation (QAM)/phase shift keying (PSK). A process of the OTFS modulation given an input codeword is described below. For ease of explanation, we define the nomenclature that *M*, *N*, Δf, and *T* mean the number of subcarriers, the number of OFDM symbols, subcarrier spacing, and OFDM symbol duration. *M* and *N* correspond to spreading length in time and frequency in case of spreading into the whole resources.

Data symbols constituting a codeword are mapped to DD resources where a 2D rectangular grid with delay spacing $1/M\Delta f$ and Doppler spacing $1/NT$ is assumed. If real channel estimation is considered, pilot symbols can be multiplexed with data symbols in an orthogonal manner in DD domain [5]. Since the scope of this paper is limited to channel equalization itself with ideal channel estimation, the use of pilot symbols are not taken into account. After the resource mapping in DD domain, the data symbols are precoded with inverse discrete symplectic Fourier transform (IDSFT), possibly followed by 2D windowing. Next, the precoded data symbols are mapped to TF resources where a 2D rectangular grid is assumed with frequency spacing Δf and time spacing *T*. It is noted that the cascaded operations by precoding followed by resource mapping in TF domain is corresponding to spreading from DD domain to TF domain. The spread data symbols placed in TF domain are OFDM-modulated, for each OFDM symbol within a transmission time interval (TTI), by taking inverse discrete Fourier transform (IDFT) followed by cyclic prefix (CP) insertion. In a consolidated form, a transmission signal matrix **S** of dimension $(M+M_{CP})\times N$ where column and row indices are associated with an OFDM symbol index and a time sample index in an OFDM symbol, respectively, after the OTFS modulation given an input codeword vector **x** of length $K(:= MN)$, is represented by

$$\mathbf{S} := \mathbf{C}_{\mathrm{I}}\mathbf{F}_M^H\mathbf{F}_M\mathbf{X}\mathbf{F}_N^H = \mathbf{C}_{\mathrm{I}}\mathbf{X}\mathbf{F}_N^H, \tag{1}$$

where $\mathbf{X} := \mathrm{vec}^{-1}_{M\times N}(\mathbf{x})$, $\mathbf{F}_J$ is a normalized $J(\in \mathbb{N})$-point DFT matrix, and $\mathbf{C}_{\mathrm{I}}$ is an $(M+M_{CP})\times M$ CP insertion matrix given CP length M_{CP}. By vectorization of input and output matrices, (1) can be altered as

$$\mathbf{s} := \mathrm{vec}(\mathbf{S}) = \underbrace{\left(\mathbf{I}_N \otimes \mathbf{C}_{\mathrm{I}}\mathbf{F}_M^H\right)}_{\substack{\text{OFDM modulation for}\\ \text{each OFDM symbol}}}\underbrace{\left(\mathbf{F}_N^H \otimes \mathbf{F}_M\right)}_{\text{IDSFT}}\mathbf{x} = \left(\mathbf{F}_N^H \otimes \mathbf{C}_{\mathrm{I}}\right)\mathbf{x}. \tag{2}$$

B. Time-Varying Multipath Fading Channels

A length-$(M+M_{\mathrm{CP}})N+L$ received signal vector **r** being captured during the TTI (i.e., *N* OFDM symbols) is expressed as

$$\mathbf{r} := \mathbf{H}_{\mathrm{T}}\mathbf{s} + \mathbf{z}_{\mathrm{T}}, \tag{3}$$

where $\mathbf{H}_{\mathrm{T}}$ is a channel impulse response (CIR) (or time-varying convolution) matrix with the maximum delay given by *L*-th sample and $\mathbf{z}_{\mathrm{T}}$ is a complex white Gaussian noise vector with $\mathrm{E}[\mathbf{z}_{\mathrm{T}}] = \mathbf{0}_{(M+M_{\mathrm{CP}})N\times 1}$ and $\mathrm{E}[\mathbf{z}_{\mathrm{T}}\mathbf{z}_{\mathrm{T}}^H] = n_0\mathbf{I}_{(M+M_{\mathrm{CP}})N}$.

C. OTFS Demodulation

The received signals suffering from time-varying multipath fading and additive white Gaussian noise undergo OTFS demodulation in a reverse order of OTFS modulation. For each OFDM symbol, the received signals are OFDM-demodulated by taking discrete Fourier transform (DFT) following CP removal. The OFDM-demodulated signals are demapped from TF resources, and then they are deprecoded with discrete symplectic Fourier transform (DSFT), possibly following 2D windowing. Similarly with the spreading in OTFS modulation, the consecutive operations by deprecoding following resource demapping in TF domain can be seen as despreading from TF domain to DD domain. Finally, the despread signals are demapped from DD resources before (or after) channel equalization. The channel equalization will be presented in Section III. In a nutshell, the OTFS-demodulated signal matrix $\tilde{\mathbf{X}}$ of dimension $M\times N$, being processed the operations above from the received signal vector **r**, in order, is represented by

$$\tilde{\mathbf{X}} := \mathbf{F}_M^H\mathbf{F}_M\mathbf{C}_{\mathrm{R}}\mathbf{R}\mathbf{F}_N = \mathbf{C}_{\mathrm{R}}\mathbf{R}\mathbf{F}_N, \tag{4}$$

where $\mathbf{R} := \mathrm{vec}^{-1}_{(M+M_{\mathrm{CP}})\times N}(\mathbf{r})$ and $\mathbf{C}_{\mathrm{R}}$ is an $M\times(M+M_{CP})$ CP removal matrix discarding the first M_{CP} samples in each OFDM symbol. (3) can be written with vectorizing input and output matrices as

$$\tilde{\mathbf{x}} := \mathrm{vec}(\tilde{\mathbf{X}}) = \underbrace{\left(\mathbf{F}_N \otimes \mathbf{F}_M^H\right)}_{\text{DSFT}}\underbrace{\left(\mathbf{I}_N \otimes \mathbf{F}_M\mathbf{C}_{\mathrm{R}}\right)}_{\substack{\text{OFDM demodulation for}\\ \text{each OFDM symbol}}}\mathbf{r} = \left(\mathbf{F}_N \otimes \mathbf{C}_{\mathrm{R}}\right)\mathbf{r}. \tag{5}$$

By substituting (2) and (3) into (4), (4) can be rearranged as

$$\tilde{\mathbf{x}} := \left(\mathbf{F}_N \otimes \mathbf{C}_\text{R}\right)\left(\mathbf{H}_\text{T}\left(\mathbf{F}_N^H \otimes \mathbf{C}_\text{I}\right)\mathbf{x} + \mathbf{z}_\text{T}\right) \\ = \mathbf{H}\mathbf{x} + \mathbf{z}, \tag{6}$$

where $\mathbf{H}\left(:= (\mathbf{F}_N \otimes \mathbf{I}_M)(\mathbf{I}_N \otimes \mathbf{C}_\text{R})\mathbf{H}_\text{T}(\mathbf{I}_N \otimes \mathbf{C}_\text{I})(\mathbf{F}_N^H \otimes \mathbf{I}_M)\right)$ is an $MN \times MN$ DD-domain effective channel matrix and $\mathbf{z}\left(:= (\mathbf{F}_N \otimes \mathbf{C}_\text{R})\mathbf{z}_\text{T}\right)$ is a length-MN DD-domain effective noise vector. It is noticed that $\mathbf{H}$ is a block circulant matrix (by its property that it is decomposed into $(\mathbf{F}_N \otimes \mathbf{I}_M)(\cdot)(\mathbf{F}_N^H \otimes \mathbf{I}_M)$ or $(\mathbf{F}_N^H \otimes \mathbf{I}_M)(\cdot)(\mathbf{F}_N \otimes \mathbf{I}_M)$) made up with N submatrices of dimension $M \times M$ and each of the submatrices is a quasi-circulant matrix (or a perfect circulant matrix if block fading). Such a structure can be viewed as a 2D (quasi-)circular convolution. Also, $\mathbf{z}$ has the same statistical properties as $\mathbf{z}_\text{T}$ except for the length (i.e., $\text{E}[\mathbf{z}] = \mathbf{0}_{K\times 1}$ and $\text{E}[\mathbf{z}\mathbf{z}^H] = n_0\mathbf{I}_K$).

III. Turbo Equalization with Damping

In this section, an MMSE-based turbo equalization in DD domain and its equalized data symbols demapping are gone through, and then two types of damping methods on top of the turbo equalization are proposed.

In the turbo equalization, the interleaved version of extrinsic LLRs of the coded bits, fed from the channel decoder, are utilized as *a priori* probabilities (or LLRs) to cancel IDSI, to suppress residual IDSI, and to demap the equalized data symbols (i.e., calculate the extrinsic LLRs of the coded bits comprising each equalized data symbol). It is noted that, at the very first iteration without *a priori* probabilities (or LLRs) fed from the channel decoder, 0 and 1 are assumed to be equiprobable for all the coded bits, and all the candidate modulation symbols for all the data symbols, too. The extrinsic LLRs of the coded bits fed from the equalization are deinterleaved and then are input to the channel decoder followed by channel decoding. Such turbo iteration is repeated until decoding success (confirmed by the CRC) or the maximum number of iterations.

As elucidated in [4], the $k(\in[K])$-th equalized data symbol is given by

$$\hat{x}_k := w_k \mathbf{h}_k^H \mathbf{A}^{-1}\left(\tilde{\mathbf{x}} - \mathbf{H}\text{E}[\mathbf{x}] + \text{E}[x_k]\mathbf{h}_k\right), \tag{7}$$

where $\hat{\mathbf{x}} := \text{vec}\left(\hat{\mathbf{X}}\right)$, $\hat{\mathbf{X}}$ is an equalized data symbol matrix, $\text{E}[\mathbf{x}] := [\text{E}[x_0] \quad \text{E}[x_1] \quad \cdots \quad \text{E}[x_{K-1}]]^T$, $\text{E}[x_k](:= \sum_{\xi\in\Xi}\Pr(x_k = \xi)\xi)$ is the mean of k-th data symbol, $\mathbf{A} := \mathbf{H}\text{C}[\mathbf{x}]\mathbf{H}^H + n_0\mathbf{I}_K$, $\text{C}[\mathbf{x}]\left(:= \text{E}[\mathbf{x}\mathbf{x}^H] - \text{E}[\mathbf{x}]\text{E}[\mathbf{x}^H] = \text{diag}\left(\text{C}[x_0], \text{C}[x_1], \ldots, \text{C}[x_{K-1}]\right)\right)$ is the covariance matrix of $\mathbf{x}$, $\text{C}[x_k](:= \text{E}[x_k x_k^*] - |\text{E}[x_k]|^2 = \sum_{\xi\in\Xi}\Pr(x_k = \xi)|\xi|^2 - |\text{E}[x_k]|^2)$ is the variance of k-th data symbol, Ξ is a (QAM/PSK) modulation alphabet. $\Pr(x_k = \xi)$ is defined by

$$\Pr(x_k = \xi) := \prod_{j\in[B]} 0.5\left(1 + \bar{b}_{\xi,j} \cdot \tanh(L_{k,j}^\text{DEC}/2)\right), \tag{8}$$

where $B := \log_2 Q$, $Q := |\Xi|$, $\bar{b}_{\xi,j} := 1 - 2b_{\xi,j}$ is $j(\in [B])$-th bit mapped to a candidate modulation symbol $\xi(\in \Xi)$, and $L_{k,j}^\text{DEC}$ is *a priori* LLR of j-th bit mapped to k-th data symbol x_k , which is fed from the channel decoder followed by the interleaver.

The equalized data symbols need to be demapped to derive the extrinsic LLRs of the coded bits thereof, which are input to a soft-input soft-output (SISO) channel decoder. In order to calculate the extrinsic LLRs of the coded bits of x_k's, a likelihood metric for the equalized data symbols is to be defined. It is a simple but practical way to approximate the likelihood function as Gaussian distribution taken into account in [4] as

$$p(\hat{x}_k \mid x_k = \xi) \approx \psi_{\mu_{k,\xi},\sigma_k^2}(\hat{x}_k), \tag{9}$$

where $\psi_{\mu,\sigma^2}(\cdot)$ is a probability density function of the Gaussian distribution with mean μ and variance σ^2 , $\mu_{k,\xi} := \xi \cdot w_k \cdot \alpha_k$, $\sigma_k^2 := w_k^2 \cdot \alpha_k(1 - \text{C}[x_k] \cdot \alpha_k)$. At the very first turbo iteration, $\mu_{k,\xi}$ and σ_k^2 are set to 0 and 1, respectively, by the fact that all the candidate modulation symbols in the modulation alphabet are equiprobable. Based on the likelihood function with the Gaussian approximation, we arrive at the extrinsic LLR of $j(\in[B])$-th bit mapped to $k(\in[K])$-th data symbol being calculated as

$$L_{k,j}^\text{EQ} := \ln\sum_{\xi|_{b_{\xi,j}=0}\in\Xi}\exp\left(\lambda_{k,j,\xi}\right) - \ln\sum_{\xi|_{b_{\xi,j}=1}\in\Xi}\exp\left(\lambda_{k,j,\xi}\right) \\ \approx \max_{\xi|_{b_{\xi,j}=0}\in\Xi}\lambda_{k,j,\xi} - \max_{\xi|_{b_{\xi,j}=1}\in\Xi}\lambda_{k,j,\xi}, \tag{10}$$

where $\lambda_{k,j,\xi} := -\left|\hat{x}_k - \mu_{k,\xi}\right|^2/\sigma_k^2 + 0.5\cdot\sum_{l\in[B]\setminus\{j\}}\bar{b}_{\xi,l}\cdot L_{k,l}^\text{DEC}$.

Next, we take deep dive into two proposed damping methods, being added on the turbo equalization and its equalized data symbol demapping accounted for above.

A. Equalizer-Fed Extrinsic LLR Damping

The first approach is to dampen the *equalizer-fed* extrinsic LLRs. The dampened *equalized-fed* extrinsic LLR at $t(\geq 1)$-th turbo iteration, $L_{k,j}^\text{EQ}(t)$, is given as follows

$$L_{k,j}^\text{EQ}(t) := \delta_\text{EQ}\cdot L_{k,j}^\text{EQ} + (1-\delta_\text{EQ})\cdot L_{k,j}^\text{EQ}(t-1), \tag{11}$$

where $L_{k,j}^\text{EQ}(0) = 0$ and δ_EQ is a damping factor for the *equalizer-fed* extrinsic LLR damping. This is a form of the first-order moving average, which controls the amount of updating a newly computed extrinsic LLR value at the current turbo iteration on top of the previously updated value. Since the post-processed residual IDSI is underestimated by the Gaussian approximation of the likelihood function when calculating the extrinsic LLR value $L_{k,j}^\text{EQ}$ before damping, its reliability may be overestimated. It is conjectured that the drastic change in the equalizer-fed extrinsic LLR is harmful at the decoding stage by being fallen into the erroneous codeword.

B. Decoder-Fed Extrinsic LLR Damping

The other approach is to dampen the *decoder-fed* extrinsic LLR damping. Specifically, *a priori* probabilities of the coded bits constituting each data symbol at $t(\geq 2)$-th turbo iteration,

TABLE I. SYSTEM PARAMETERS

Parameter	Value
Carrrier frequency	28 GHz
Subcarrier spacing	480 kHz
System bandwidth	50 MHz
FFT size	128
CP length	16 samples
# of allocated subcarriers (M)	32
Transmission time interval (N)	12 OFDM symbols
2D spreading length (K)	384 (=32×12)
Data symbol modulation scheme	64-QAM with Gray mapping
Channel code	5G NR LDPC
Code rate	772/1024
Transport block size	1736 bits
CRC size	16 bits
Channel estimation	Ideal
Data symbol demapper	Max-log-MAP
Channel decoder	Normalized min-sum (scaling factor: 0.75)
# of antenna ports at TX/RX	1T1R

$\Pr(x_k = \xi; t)$, need to be obtained by dampening them when reckoning the mean and variance of the data symbols as follows

$$\Pr(x_k = \xi; t) := \delta_{\mathrm{DEC}} \cdot \Pr(x_k = \xi) + (1 - \delta_{\mathrm{DEC}}) \cdot \Pr(x_k = \xi; t-1), \quad (12)$$

where $\Pr(x_k = \xi; t = 1) = 0.5$ and δ_{DEC} is a damping factor for the *decoder-fed* extrinsic LLR damping. In the MMSE-based turbo equalization, this approach plays a role in more gradually reconstructing the soft data symbol (i.e., mean) to cancel out and to suppress the residual IDSI after such a milder soft cancelation since the *a priori* probabilities induced from the *decoder-fed* extrinsic LLRs after passing through the interleaver are updated smoothly by the damping thereof.

IV. LINK-LEVEL PERFORMANCE EVALUATIONS

In this section, we conduct link-level performance evaluations in terms of SNR vs. BLER to demonstrate the efficacy of the two proposed damping methods.

A. Evaluation Assumptions

For the link-level performance evaluations, the evaluation parameters are configured as in Table I. It is noted that subcarrier spacing is set to 480 kHz to allow inter-carrier interference negligibly small (or to meet a block fading constraint almost). 5G NR LDPC is employed for a channel code [6] and 16-bit CRC is used. 32 subcarriers out of 128 subcarriers are allocated for the OTFS modulation.

In order to generate time-varying multipath fading channels, a channel coefficient of each path is randomly generated by Rayleigh distribution with the same mean (i.e., uniform channel gain in an average sense). There consider three channel scenarios with respect to the number of paths and a combination of delay and Doppler shift for each path, as provided in TABLE II. The maximum Doppler frequency $f_{\mathrm{D}}^{\max}$ is set to 12.963 kHz, which is corresponding to 500 km/h given the carrier frequency, and its normalized version $f_{\mathrm{D}}^{\max} \cdot T$ is 0.030.

TABLE II. CHANNEL-RELATED PARAMETERS

Channel Scenario	The number of paths	Delay [samples]	Doppler shift (normalized by max. Doppler frequency)
A	4	{0, 8, 0, 8}	{+1, +0.5, -1, -0.5}
B	6	{0, 6, 12, 0, 6, 12}	{+1, +0.5, +0.25, -1, -0.5, -0.25}
C	8	{0, 4, 8, 12, 0, 4, 8, 12}	{+1, +0.5, +0.25, +0.125, -1, -0.5, -0.25, -0.125}

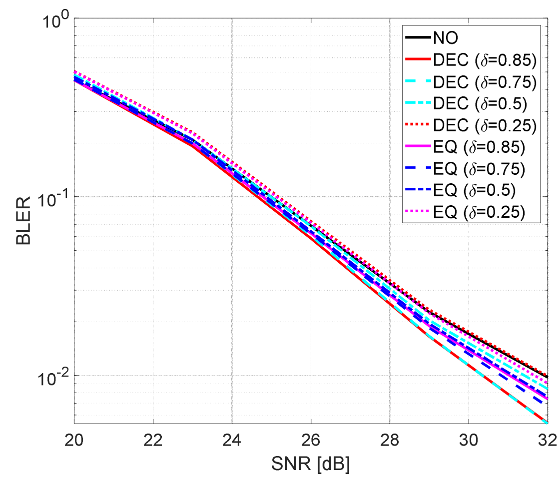

Fig. 1. SNR vs. BLER performance comparisons of the damping methods with different damping factors.

B. Evaluation Results

Fig. 1 depicts the SNR vs. BLER performance for three types of damping: 1) no damping (shortly called NO), 2) equalizer-fed extrinsic LLR damping (shortly called EQ-fed damping or EQ), and 3) *decoder-fed* extrinsic LLR damping (shortly called DEC-fed damping or DEC) for Channel Scenario A. For the EQ-fed and DEC-fed damping methods, four values of the damping factor (i.e., $\delta \in$ {0.25, 0.5, 0.75, 0.85}) are configured to verify how it can affect the performance and to select the best one. It is noted that no damping is equivalent to a damping factor being set to 1 for both of the DEC- and EQ-fed damping methods. We can observe that both the damping methods outperform the case without damping, and the DEC-fed damping is superior to the EQ-fed damping under their damping factors selected in having the best performance (say, 0.85 and 0.75 for DEC- and EQ-fed damping, respectively). In the case of the DEC-fed damping, the performance is getting better as the larger damping factor is used while EQ-fed damping shows the best performance when the damping factor is set to 0.75. Also, the performance deviation of the EQ-fed damping with respect to the damping factor is smaller than that of the DEC-fed damping. It is worthwhile to note that the efficacy of damping becomes noticeable as the SNR is increased since the post-processed residual IDSI is dominant at the high SNR regime. The best SNR gain is given by approximately 1.6 dB at BLER 10^{-2} when employing the DEC-fed damping with a damping factor of 0.85.

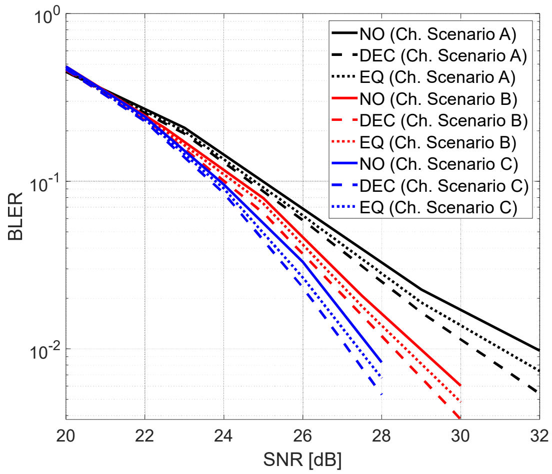

Fig. 2. SNR vs. BLER performance comparisons of the damping methods for different channel scenarios.

Fig. 2 draws the SNR vs. BLER performance in three channel scenarios with respect to the three types of damping (i.e., NO, EQ, and DEC) to observe whether the DEC- and EQ-fed damping methods work better than the case without damping for different channel environments or not. The damping factors are set to 0.85 and 0.75 for the DEC- and EQ-fed damping methods, respectively. For Channel Scenario A (with four channel paths), we observe SNR gains of 1.6 dB and 1 dB for the DEC- and EQ-fed damping methods compared to the no damping, respectively, while they become small as 0.7 dB and 0.4 dB, respectively, for Channel Scenario B (with six channel paths). In the case of Channel Scenario C (with eight channel paths), SNR gains look marginal as 0.5 dB and 0.3 dB for the DEC- and EQ-fed damping methods in comparison to the no damping, respectively. Such phenomena are because the Gaussian approximation on the likelihood function in terms of the post-processed residual IDSI becomes more inaccurate as the channel paths are getting sparser by the central limit theorem. Furthermore, it is found that DEC-fed damping has superiority over EQ-fed damping in all the channel scenarios conducted here.

V. Conclusion

In this paper, we proposed two damping methods: 1) *equalizer-fed* extrinsic LLR damping and 2) *decoder-fed* extrinsic LLR damping for the MMSE-based turbo equalization in the coded OTFS system. The link-level performance evaluations have shown that they were effective to improve SNR vs. BLER performance at a very marginal expense of additional computation in the damping, merely employing the same equalization and channel decoder. Furthermore, it was observed that the *decoder-fed* extrinsic LLR damping provided better link-level performance than that of the *equalizer-fed* extrinsic LLR damping. In our future work, we will go theoretically deep into the study of how such damping methods improve the link-level performance.

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