A Common Phase Error Estimation Scheme with MMSE Equalizer

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***Abstract*—Phase noise becomes a critical problem as wireless communication systems utilize high frequency band such as mmWave or THz. The effects of phase noise should be correctly estimated and compensated to achieve the required performance. In this paper, a linear MMSE based common phase error estimation algorithm is proposed under 3GPP NR DM-RS and PT-RS structure. It is shown that the proposed algorithm can estimate common phase error efficiently with reduced complexity.**

***Index Terms*—Phase noise, common phase error, MMSE equalizer**

I. Introduction

OFDM is widely adopted as a modulation scheme in broadband communication systems based on its robustness to multi-path fading and bandwidth efficiency. To address the increasing data rate requirements, wireless communication systems are seeking high frequency bands such as mmWave or THz to utilize wide bandwidth available and also endorse high modulation orders such as 256- or 1024-QAM. As the carrier frequency and modulation order get higher, the effects of phase noise become severe [1].

Phase noise is incurred from local oscillators (LO) since the output of the LO is not perfectly aligned to the carrier frequency but continuously fluctuates in the vicinity of the target frequency. The phase noise induces frequency offset and it destructs the orthogonality among the subcarriers. Deteriorating effects of the phase noise can be categorized into two: common phase error (CPE) and inter-carrier interference (ICI). Within an OFDM symbol, the CPE is common to all subcarriers and rotates the whole signal constellation in one direction. On the other hand, the ICI induces interference from other subcarriers.

From the first release of 3GPP NR, it has incorporated phase-tracking reference signal (PT-RS) in addition to legacy demodulation reference signal (DM-RS) [2]. If configured, the PT-RS is transmitted with DM-RS to figure out the effects of phase noise within scheduled resources. One limitation of the NR PT-RS is that it is designed only to esimate CPE [3].

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In this paper, we propose a novel CPE estimation algorithm based on 3GPP NR DM-RS and PT-RS structure. Since DM-RS and PT-RS are allocated to only few resource elements (RE) within a scheduled resource and two reference signals are not transmitted simultaneously, it is essential to develop a scheme to estimate channel and CPE at each symbol. The proposed algorithm estimates the difference between CPE estimates from DM-RS and PT-RS with linear MMSE equalizer. With the identified differences, a channel estimates with correct CPE estimates can be obtained.

II. System Model

A. Received Signal Model with Phase Noise

A baseband representation of the OFDM modulator output can be written as

$$x_n(l) = \sum_{k=0}^{N-1} s_k(l) e^{j\frac{2\pi nk}{N}}, \quad n = 0, 1, \cdots, N-1 \tag{1}$$

where $s_k(l)$ is the transmit symbol at the k-th subcarrier in the OFDM symbol l. The baseband signal is converted into analog signal using a pulse shaping filter $g(t)$ and transmit signal is given by

$$x(t) = \sum_{n=0}^{N-1} x_n(l) g(t - nT), \quad 0 \le t < T_s \tag{2}$$

where T_s represents symbol duration with $T_s = NT$ and T denotes sampling period. The transmitted OFDM signal $x(t)$ is convolved with channel and the received with phase and thermal noise. It can be represented by

$$z(t) = e^{j\phi(t)} \cdot (h(t) * x(t)) + w(t). \tag{3}$$

Here, $\phi(t)$ represents the phase noise. Generally, the phase noise $\phi(t)$ of a free running oscillator is modelled by a Wiener process while phase-locked oscillators are modelled using a filtered Gaussian process with a power spectral density [4].

The received signal (3) is then sampled with period T and FFT is performed after CP removal. Frequency domain representation of (3) at subcarrier n of an OFDM symbol l is

$$Z_n(l) = J_0(l) H_n(l) s_n(l) + \underbrace{\sum_{\substack{m=0,\\ m\neq n}}^{N-1} J_{n-m}(l) H_m(l) s_m(l) + w_n(l)}_{\triangleq w'_n(l)} \tag{4}$$

where $H_n(l)$ is frequency domain channel response at subcarrier n and $J_k(l)$ is given by

$$J_k(l) = \frac{1}{N}\sum_{n=0}^{N-1} e^{j\left(\phi_n - \frac{2\pi nk}{N}\right)} \quad (5)$$

with $\phi_n = \phi(nT)$. It can be noted from (4) that for each subcarrier n, the common CPE term, $J_0(l)$, is multiplied to the desired received signal and hence, the CPE rotated received signal according to $\arg(J_0(l))$; the ICI term can be merged with thermal nosie and that is represented with $w'_n(l)$ from now on.

B. 3GPP NR DM-RS and PT-RS Structure

3GPP NR provides various reference signals designed for own purposes. DM-RS and PT-RS are two reference signals for channel estimation and phase noise tracking [2]. To reduce the overhead, DM-RS is allocated sparsely in both frequency and time domain. Within a slot, DM-RS is transmitted in one to four symbols according to network configuration. Due to its sparsity in time and frequency domain, channel estimation for a scheduled resource should adopt some kind of interpolation method among channel estimates on DM-RS symbols. PT-RS is even sparser than DM-RS in the frequency domain (transmitted only in every 2 or 4 RBs) while denser in time domain (every one to four symbols within a slot). These differences are based on the nature of phase noise that CPE is common in all subcarriers while the CPE generally varies according to symbol.

III. The Proposed Algorithm

A. Observaton on Channel Estimation

Assume that a data packet is transmitted over a resource grid of KL resource elements which consists of L OFDM symbols and K sub-carriers. Among the allocated L symbols, DM-RS is transmitted only over $D < L$ symbols according to higher layer configuration. Using DM-RS, we can estimate channel at the DM-RS symbol while the channel estimates at the REs without DM-RS should be acquired by inferring from the channel estimates at the DM-RS symbols.

Let l_k denote an OFDM symbol index within the scheduled resource, i.e., $0 \le l_k < L$ where DM-RS is transmitted ($k = 0, 1, \cdots, D-1$ denotes DM-RS symbol indices). Similarly, we can define $\bar{l}_k$ as symbol indinces within the scheduled resources without DM-RS (thus, $k = 0, 1, \cdots, L-D-1$). Channel estimates at an arbitrary $\bar{l}_k$ can be obtained by, for example, a linear interpolation of nearby channel estimates at DM-RS symbols. In this sense, the channel estimates consist of two groups: One is channel estimates directly acquired at the DM-RS symbol and the other is interpolated channel estimates.

Assume that the channel in the air changes slowly so that wihin a slot, $H_n(l) \approx H_n$ for $\forall l$. On the other hand, CPE $J_0(l)$, changes independently according to symbol index l. The channel estimate at a DM-RS RE can be considered as a product of the estimates of the air channel and CPE, i.e.,

$$\hat{H}_n(l_k) \triangleq \hat{J}_0(l_k)\tilde{H}_n(l_k). \quad (6)$$

Similarly, channel estimate at another DM-RS symbol, $l_{k'}$, ($k \neq k'$) can be written as

$$\hat{H}_n(l_{k'}) \triangleq \hat{J}_0(l_{k'})\tilde{H}_n(l_{k'}). \quad (7)$$

where the estimated channel at $l_{k'}$ can also be decomposed by a product of the CPE component at $l_{k'}$ and the channel estimate at l_k. Based on the assumption on the slow varying nature of the channel stated above, $\tilde{H}_n(l_k)$ can be assumed to be invariant with l_k and thus, we can omit symbol index in channel estimate $\tilde{H}_n(l_k)$ as

$$\tilde{H}_n(l_k) = \tilde{H}_n, \quad \text{for} \quad k = 0, 1, \cdots, D-1. \quad (8)$$

Based on (8), the channel estimate at a non DM-RS symbol $\bar{l}_k$ can be decomposed by

$$\hat{H}_n(\bar{l}_k) = \hat{J}_0(\bar{l}_k)\tilde{H}_n. \quad (9)$$

It is rationale that the interpolated $\hat{J}_0(\bar{l}_k)$ is different from the real CPE, $J_0(\bar{l}_k)$, since $\hat{H}_n(\bar{l}_k)$ is obtained by an interpolation of the channel estimates $\{\hat{H}_n(l_k)\}$ where CPEs at DM-RS symbols are not correlated with CPE at $\bar{l}_k$. Therefore, to figure out $J_0(\bar{l}_k)$, we need to estimate difference between $\hat{J}_0(\bar{l}_k)$ and $J_0(\bar{l}_k)$.

B. Linear MMSE CPE Estimation Algorithm

Let $\hat{l}_k$ for $k = 0, 1, ..$ denote a symbol index where PT-RS is transmitted (Remark that $\hat{l}_k$ is one of $\bar{l}_k$). In frequency domain, PT-RS is transmitted on a subset $\mathcal{K}(\hat{l}_k)$ among K sub-carriers. Let $x_n(\hat{l}_k)$ be a transmitted PT-RS symbol at subcarrier $n \in \mathcal{K}(\hat{l}_k)$ on symbol $\hat{l}_k$. Then, we can generate a modified channel estimate by multiplying the channel estimate, $\hat{H}_n(\hat{l}_k)$ and the PT-RS symbol $x_n(\hat{l}_k)$ for all $n \in \mathcal{K}(\hat{l}_k)$:

$$\breve{H}_n(\hat{l}_k) \triangleq \hat{H}_n(\hat{l}_k)x_n(\hat{l}_k) = \hat{J}_0(\hat{l}_k)\tilde{H}_n x_n(\hat{l}_k). \quad (10)$$

The received signal at PT-RS RE can be extracted from (4) as

$$Z_n(\hat{l}_k) = J_0(\hat{l}_k)H_n(\hat{l}_k)x_n(\hat{l}_k) + w'_n(\hat{l}_k), \quad \forall n \in \mathcal{K}(\hat{l}_k). \quad (11)$$

Then (11) can be approximately written with (10) as

$$Z_n(\hat{l}_k) \approx \breve{H}_n(\hat{l}_k) \times (J_0(\hat{l}_k)/\hat{J}_0(\hat{l}_k)) + w'_n(\hat{l}_k). \quad (12)$$

Therefore, if we obtain linear MMSE equalizer $W_n(\hat{l}_k)$ with $\breve{H}_n(\hat{l}_k)$, an estimate on the ratio of $J_0(\hat{l}_k)$ and $\hat{J}_0(\hat{l}_k)$ can be computed. Let

$$\gamma_n(\hat{l}_k) \triangleq W_n(\hat{l}_k)Z_n(\hat{l}_k) \approx \frac{J_0(\hat{l}_k)}{\hat{J}_0(\hat{l}_k)}, \quad \forall n \in \mathcal{K}(\hat{l}_k). \quad (13)$$

Final ratio estimate of the real and estimated CPE is obtained by averaging $\{\gamma_n(\hat{l}_k)\}$:

$$\hat{\gamma}(\hat{l}_k) = \frac{1}{|\mathcal{K}(\hat{l}_k)|}\sum_{n \in \mathcal{K}(\hat{l}_k)} \gamma_n(\hat{l}_k). \quad (14)$$

where $|\cdot|$ denotes a cardinality of a set. Finally, we update the channel estimate at $\hat{H}_n(\hat{l}_k)$ by mutliplying $\hat{\gamma}(\hat{l}_k)$ for all sub-carriers. For a symbol $\bar{l}_k$ which does not correspond to $\hat{l}_k$ utilizes the most recent $\hat{\gamma}(\hat{l}_k)$.

Algorithm 1: The proposed CPE estimation algorithm

Obtain channel estimates, $\hat{H}_n(l)$, $\forall n, l$ with DM-RS;
for $l = 0$ *to* $L-1$ **do**
  if l *is not DM-RS symbol* **then**
    if l *is PT-RS symbol* **then**
      Calculate modified channel estimate, $\breve{H}_n(l) = \hat{H}_n(l)x_n(l), \forall n \in \mathcal{K}(l)$;
      Gather received signal on PT-RS RE, $Z_n(l), \forall n \in \mathcal{K}(l)$;
      Perform MMSE equalizer on $Z_n(l)$ with $\breve{H}_n(l)$ and $w'_n(l), \forall n \in \mathcal{K}(l)$;
      Compute $\hat{\gamma}(l)$ by (14);
    else
      `// Symbol w/o DM-RS and PT-RS`
      Use the previous CPE error estimate on PT-RS symbol as the final CPE ratio estimate
    end
    Multiply the final CPE ratio estimate to the channel estimate, $\hat{H}_n(l)$
  end
end

IV. Simulation Results

In this section, we will provide numerical simulation results. In this paper, we have adopted filtered Gaussian phase noise model which is a more pragmatic phase noise model provided [4].

The power spectral density of the phase noise is given by

$$S(f) = \begin{cases} S_{\text{Ref}}(f) + S_{\text{PLL}}(f), & f \leq \text{loop BW} \\ S_{\text{VCO\_V2}}(f) + S_{\text{VCO\_V3}}(f) & f > \text{loop BW} \end{cases} \tag{15}$$

where PSD of each component is

$$S_{\text{Ref/PLL/VCO\_V2/VCO\_V3}} = \text{PSD}_0 \cdot \left[\frac{1 + (f/f_z)^k}{1 + f^k} \right] \tag{16}$$

and $\text{PSD}_0 = \text{FOM} + 20 \log f_c - 10 \log(\frac{\text{P}}{1\text{mW}})$. Detailed values of the paramters can be found in [4].

In Fig. 1, the BLER performance of the proposed algorithm is shown. Some of simulation parameters adopted to obtain the simulation results are shown in Table I.

In the figure, we can see that without proper phase noise compensation, the BLER performance is severely degraded by referring the graph labeled with 'No Compensation'. The performance of the proposed algorithm is compared with a conventional CPE estimation algorithm [3]. The conventional CPE estimation algorithm is that the channel estimates using DM-RS are obtained as in our paper. In addition to that channel estimates using PT-RS are obtained and compared. The difference between them are considered to be from phase noise difference. The conventional algorithm suffers from high computational complexity since channel estimation process usually includes many steps such as denoising, interpolation, and so on. In Fig. 1, the BLER performance of the proposed algorithm is approximately 1dB worse than the conventional algorithm while the computational complexity of the proposed algorithm is less than that of the conventional algorithm.

TABLE I
Evaluation Parameters

Parameter	Value
Channel model	TDL-A [5] (Delay spread = 30ns)
Carrier frequency	30GHz
Bandwidth	100MHz
Subcarrier Spacing	120kHz
FFT size	1024
Phse noise model	UE model in chapter 6.1.11 [4]
Number of Tx/Rx antenna port	2
Number of layer	1
MCS scheme	64-QAM, R =0.554
DM-RS symbol index	2, 9
Number of scheduled symbols	10
Number of scheduled RBs	66
PT-RS configuration [2]	$L_{\text{PT-RS}} = 0, K_{\text{PT-RS}} = 1$

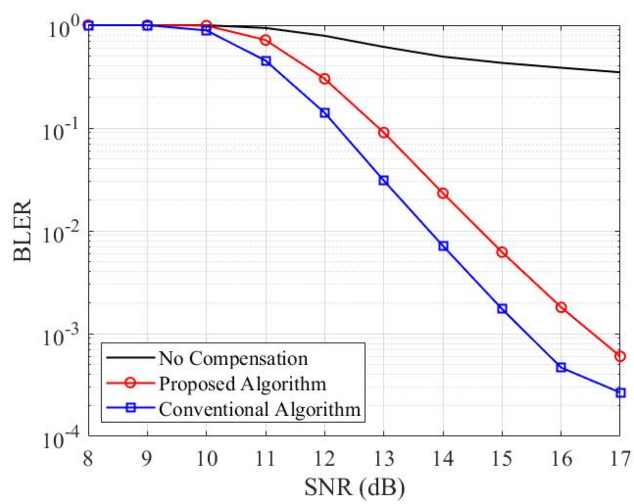

Fig. 1. BLER performance of the proposed algorithm

V. Concluding Remarks

In this paper, we have proposed a novel CPE estimation scheme. The proposed algorithm estimates the difference between the estimated CPE and real CPE by using linear MMSE equalizer. The proposed algorithm is specifically useful when the channel estimates are obtained by sparse DM-RS structure as in 3GPP NR.

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