Wireless Embedded Index Coded Transmissions with Multiple Antennas

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***Abstract*—Embedded index coding, known as device-to-device index coding, is a promising transmission scheme that effectively reduces the number of transmissions required in device-to-device communications. Leveraging spatial multiplexing gain attained with multiple antennas in wireless environment, this paper explores the benefits of spatially multiplexed embedded index codes through the joint design of embedded index coding and multicast beamformers. Through our simulations, we are able to demonstrate significant improvements obtained by the proposed scheme in terms of spatially multiplexed embedded index coding.**

I. Introduction

The exponential growth of wireless data traffic, estimated to reach 120 exabytes by 2026 [1], necessitates effective management of limited wireless resources. To address this challenge, multicasting transmission has emerged as an efficient solution for handling the increasing wireless traffic. By delivering identical data simultaneously to multiple users with the same request, multicasting minimizes resource usage. Index coding (IC) [2] utilizes exclusive-or (XOR) operations and leverages stored information at users to fulfill their requests concurrently. This technique has gained momentum due to the widespread availability of cost-effective large memory in user devices, making it a promising approach for reducing wireless data traffic. Building upon the concept of index coding, the embedded index coding (EIC) problem [3] was introduced as an extension specifically tailored for device-to-device settings. In particular, [4] presents a novel approach to the EIC problem and offers the optimal length of EIC under wired communications.

However, most research on IC and the EIC problem has been conducted under the assumption of wired transmissions, overlooking channel conditions such as fading [5]. In wired scenarios, the optimal index code is independent of channel conditions and simply corresponds to the shortest code. In contrast, [6] highlights the dependence of the optimal index code on channel conditions. However, their findings are limited to single-input single-output broadcast channels with user cache, and extending their applicability to multiple antenna setups is not straightforward. Designing spatially multiplexed index codes is particularly complex due to the NP-hard nature of finding the optimal index code. Recently, [7] investigates the benefits of spatially multiplexed index codes in multiple-input single-output broadcast channels.

This paper extends the findings of [7] to a device-to-device IC scenario, exploring the advantages of spatially multiplexed EIC in a multiple-input and multiple-output device-to-device (MIMO D2D) communication. As far as our understanding extends, this represents the pioneering endeavor to tackle the EIC problem through harnessing the spatial multiplexing gain within a wireless multi-antenna context. Our approach involves two main steps. First, we employ a conjugate gradient method to obtain the multi-group multicast beamformer for an arbitrary index code. Next, we identify feasible EICs from all possible clique cover index codes based on the cached information at users. Subsequently, we select the optimal EIC that minimizes the total transmission time, taking into account the corresponding beamformer. Through simulations, we demonstrate that our proposed scheme reduces transmission time across all signal-to-noise ratio (SNR) regions, highlighting the significant gains achieved by spatially multiplexed EICs.

Notations: $\|\mathbf{v}\|$ denote the vector norm of a vector $\mathbf{v}$. $[1 : K]$ denote the sequence of integers from 1 to K.

II. System Model and Problem Description

Consider a half-duplex MIMO D2D communication scenario where each user has a cache and is equipped with N_t antennas. In this setup, we have a file library $\mathcal{F}$ consisting of N files. These files are distributed among a set of users, denoted as $\mathcal{K}$, where the total number of users is K ($\leq N$).

Each user $k \in \mathcal{K}$ has a specific file demand $d_k \in \mathcal{F}$, which is not available in their local storage $\mathcal{M}_k \subseteq \mathcal{F}$. Therefore, in order to fulfill the file requests of each user, data transmission is required from other users who possess the requested files. This process takes into consideration the demands $\mathbf{d} = [d_1, \ldots, d_K]$ of all users and the cached information stored in $\mathcal{M}_1, \ldots, \mathcal{M}_K$. It is assumed that each file in $\mathcal{F}$ has a size of B bits, and the requested files of different users are distinct, meaning that $d_i \neq d_j$ for all $i \neq j$.

The considered scenario can be viewed as an EIC problem [3], which is further extended to the wireless environment. In the extended version of the EIC problem for the wireless environment, the focus goes beyond merely minimizing the length of the coded scheme. The goal is to minimize the transmission time of the coded scheme by exploiting the spatial multiplexing gain offered by multiple antennas at users.

In the half-duplex MIMO D2D communication, there are K users involved, and in each communication round $t \in [1:K]$, only one user can act as the sender. During round t, the sending user (referred to as user t) serves the other receiving users in $\mathcal{K}\backslash t$ by encoding the demands of the dedicated receiving users based on the cached information $\mathcal{M}_t$. It should be noted that round t can be skipped if there are no dedicated receiving users to be served by user t. Then, the encoded messages served by user t are simultaneously transmitted via spatial multiplexing. The received signal of user $k \in \mathcal{K}\backslash t$ during round t is denoted as $\mathbf{y}_k[t] \in \mathbb{C}^{N_t \times 1}$ and can be modeled as:

$$\mathbf{y}_k[t] = \mathbf{H}_k^\dagger[t]\mathbf{x}[t] + \mathbf{z}_k[t],$$

where $\mathbf{H}_k[t] \in \mathbb{C}^{N_t \times N_t}$ is user k's channel matrix at communication round t whose element is a complex Gaussian random variable with zero mean and unit variance, i.e., $\mathbf{H}_k[t] \sim \mathcal{CN}(0, \mathbf{1}_{N_t \times N_t})$, and $(\cdot)^\dagger$ denotes the conjugate transpose of a vector. We consider a slow fading channel model so that the channel coefficient $\mathbf{H}_k[t]$ is a constant during a single transmission period but varies across different transmissions. It is also assumed that the transmitter exactly knows the users' channels, i.e., $\{\mathbf{H}_1[t], \ldots, \mathbf{H}_K[t]\}$. Also, $\mathbf{x}[t] \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{z}_k[t] \in \mathbb{C}^{N_t \times 1}$ are a transmit signal sent by user t and a complex Gaussian noise with zero mean and unit variance, respectively. The transmission is comprised of two components: 1) the EIC and 2) beamforming vectors. Define $\mathcal{E} \triangleq \{m_1, \ldots, m_l\}$ as the set of encoded messages that satisfy all the demands $\mathbf{d}$ of users (i.e., $l \leq K$). Since some messages are sent simultaneously via spatial multiplexing, thereby the required transmission rounds can be less than l and K.

Now, we can represent an EIC with K disjoint groups, where each group corresponds to the subset of encoded messages to be transmitted by each sending user during each communication round. Let $\mathcal{G}_t \subset \mathcal{E}, \forall t \in [1:K]$ be the set of encoded messages that are sent simultaneously by user t. Then, $\mathcal{G}_1 \cup \ldots \cup \mathcal{G}_K = \mathcal{E}$. As aforesaid, it is possible that the number of required transmission rounds is less than K, resulting from some groups being empty sets. That is, the transmit signal $\mathbf{x}[t] \in \mathbb{C}^{N_t \times 1}$ at communication round t is constructed as follows:

$$\mathbf{x}[t] \triangleq \sum_{i \in \mathcal{I}_t} \mathbf{v}_i m_i, \tag{1}$$

where $\mathcal{I}_t$ is the set of indices of the encoded messages in $\mathcal{G}_t$, $\mathbb{E}[m_i] = 0$ and $\mathbb{E}[|m_i|^2] = 1, \forall i \in [1:l]$. $\mathbf{v}_i \in \mathbb{C}^{N_t \times 1}$ is a beamformer for the encoded message m_i such that $\sum_{i \in \mathcal{I}_t} \|\mathbf{v}_i\|^2 \leq P$ where P is the total transmit power budget for each user. Here, we independently encode (XORed) bit streams into encoded message and the demand of each user is associated with no more than one encoded messages in $m_1, \ldots, m_l$, thus $\mathbb{E}[\|\mathbf{x}[t]\|^2] \leq P$. Note that, if some users are served together, there exists an empty group, and we do not allocate power to the corresponding beam, i.e., $\|\mathbf{v}_i\|^2 = 0$ whenever $\mathcal{G}_i = \emptyset$, implying zero transmission time.

For a given EIC $\mathcal{G}_\mathcal{K}(\triangleq \mathcal{G}_1, \ldots, \mathcal{G}_K)$ dedicated to the users in $\mathcal{K}$, at each communication round t, the transmitter t (i.e., user t) allocates transmit power P to $|\mathcal{G}_t|$ different streams (or $|\mathcal{G}_t|$ beamforming vectors) to minimize the transmission time. Denote $\mathcal{D}_t \subset \mathcal{K}$ as the set of users served at communication round t, and $\mathcal{C}(\cdot)$ returns the index of the encoded message that is required by the receiving user. Then, receiving user k's signal-to-interference-plus-noise-ratio (SINR) becomes the function of beamforming vectors $\mathbf{v}_{\mathcal{I}_t}(\triangleq \mathbf{v}_i, \forall i \in \mathcal{I}_t)$ given by

$$\mathsf{SINR}_{tk}(\mathbf{v}_{\mathcal{I}_t}) \triangleq \frac{\|\mathbf{H}_k^\dagger[t]\mathbf{v}_{\mathcal{C}(k)}\|^2}{\sum_{i \in \mathcal{I}_t \backslash \{\mathcal{C}(k)\}} \|\mathbf{H}_k^\dagger[t]\mathbf{v}_i\|^2 + 1}, \tag{2}$$

and the receiving user k's achievable rate is $R_{tk} \triangleq W \log(1 + \mathsf{SINR}_{tk}(\mathbf{v}_{\mathcal{I}_t}))$, where W is total system bandwidth. Thus, the transmission time required by transmitter t (i.e., sending user t) becomes B/R_{tk}.

Since transmitter t serves all encoded messages in $\mathcal{G}_t$ by spatial multiplexing at communication round t, the total transmission time required to serve all the users becomes

$$T \triangleq \sum_{t=1}^{K} T_t = \sum_{t=1}^{K} \max_{k \in \mathcal{D}_t} \left[\frac{B/W}{\log(1 + \mathsf{SINR}_{tk}(\mathbf{v}_{\mathcal{I}_t}))} \right].$$

Notice that the total transmission time is affected by the constructed code design $\mathcal{G}_t$ and their beamformer $\mathbf{v}_{\mathcal{I}_t}$ for all communication round $t \in [1:K]$.

Thus, we formulate the problem that finds the optimal EIC and the optimal beamforming vectors to minimize the total transmission time as follows:

$$\begin{aligned}
(\mathsf{P1}) \quad & \underset{\substack{(\mathcal{G}_1, \ldots, \mathcal{G}_K), \\ (\mathbf{v}_{\mathcal{I}_1}, \ldots, \mathbf{v}_{\mathcal{I}_K})}}{\text{minimize}} \quad T \\
& \text{subject to} \quad \sum_{i \in \mathcal{I}_t} \|\mathbf{v}_i\|^2 \leq P, \quad \text{for all} \quad t \in [1:K], && (3) \\
& \qquad \mathcal{G}_1 \cup \ldots \cup \mathcal{G}_K = \mathcal{E}, \quad \text{where} \quad \mathcal{E} \in \mathscr{P}, && (4) \\
& \qquad \mathcal{G}_i \cap \mathcal{G}_j = \emptyset \quad \text{for all} \quad i \neq j, && (5)
\end{aligned}$$

where $\mathscr{P}$ is the all possible set of encoded messages.

III. Joint Design of EIC and Beamformer

Since the problem ($\mathsf{P1}$) cannot be directly solved, we solve ($\mathsf{P1}$) iteratively. First, we design the beamforming vectors for an arbitrarily given EIC and then exhaustively search the optimal EIC with the proposed beamformer. For an arbitrarily given $\mathcal{G}_1, \cdots, \mathcal{G}_K$, the beamformers $\mathbf{v}_{\mathcal{I}_t}$ at communication round t, minimizing the transmission time T_t, are independent of the other communication rounds $[1:K] \setminus \{t\}$, due to half-duplex transmission. Moreover, the transmission time at each communication round is a monotonic decreasing function of SINR. As a result, given $\mathcal{G}_1, \cdots, \mathcal{G}_K$, the problem ($\mathsf{P1}$) is equivalently resolved by optimizing the subproblems ($\mathsf{SP1}$) for every communication round $t \in [1:K]$,

$$\begin{aligned}
(\mathsf{SP1}) \quad & \underset{\mathbf{v}_{\mathcal{I}_t}}{\text{maximize}} \quad \min_{k \in \mathcal{D}_t} \mathsf{SINR}_{tk}(\mathbf{v}_{\mathcal{I}_t}) && (6) \\
& \text{subject to} \quad (3).
\end{aligned}$$

Since the beamforming vectors $\mathbf{v}_{\mathcal{I}_t}$ is for the encoded messages in the group $\mathcal{G}_t$, the problem ($\mathsf{SP1}$) can be regarded as a general multicast beamforming problem, which is known as NP-hard [8]. Therefore, we propose a conjugate gradient based multi-group multicast beamformer (CGB) [7], [9].

A. Description of CGB

Since the procedure is the same for all communication round $[1:K]$, here we focus on the subproblem $(\mathsf{SP1})$ at round t. The minimum SINR in (6) is the same as

$$\min_{k\in\mathcal{D}_t}\left[\|\mathbf{H}_k^\dagger[t]\mathbf{v}_{\mathcal{C}(k)}\|^2\Big/\left(\sum_{i\in\mathcal{I}_t\setminus\{\mathcal{C}(k)\}}\|\mathbf{H}_k^\dagger[t]\mathbf{v}_i\|^2+1\right)\right].$$

Thus, to apply the gradient descent algorithm, $(\mathsf{SP1})$ is transformed into an equivalent parametric programming problem with an auxiliary parameter η [10]. Thus, we transform the objective into $\mathsf{F}(\mathbf{v}_{\mathcal{I}_t},\eta)\triangleq\min_k f_k(\mathbf{v}_{\mathcal{I}_t},\eta)$, where

$$f_k(\mathbf{v}_{\mathcal{I}_t},\eta)\triangleq\|\mathbf{H}_k^\dagger[t]\mathbf{v}_{\mathcal{C}(k)}\|^2-\eta\cdot\left(\sum_{i\in\mathcal{I}_t\setminus\{\mathcal{C}(k)\}}\|\mathbf{H}_k^\dagger[t]\mathbf{v}_i\|^2+1\right).$$

Due to a non-smoothness of $\mathsf{F}(\mathbf{v}_{\mathcal{I}_t},\eta)$, using the log-exp smoothing function, we change the objective functionand rewrite the problem $(\mathsf{SP1})$ as

$$(\mathsf{SP2})\quad \underset{\mathbf{v}_{\mathcal{I}_t}}{\text{maximize}}\quad \mu\log\left(\textstyle\sum_{k\in\mathcal{D}_t}\exp(-f_k(\mathbf{v}_{\mathcal{I}_t},\eta)/\mu)\right)$$
$$\text{subject to}\quad (3).$$

Here, μ is a smoothing parameter. We can solve the problem $(\mathsf{SP2})$ by using the Dinkelbach-type Riemannian conjugate gradient (DT-RCG) algorithm [9].

B. Optimal EIC search

To obtain the optimal solution for the problem $(\mathbf{P1})$, the straightforward approach is to exhaustively search all the EICs by solving the subproblem $(\mathsf{SP2})$ for all communication round $t\in[1:K]$. Since the encodable/decodable EIC set cannot be structured, the first step is to identify all feasible EICs among the potential options for a given cached information $\mathcal{M}$. Subsequently, we iteratively compare the total transmission time T for the feasible EICs to determine the best solution.

Due to the absence of closed-form solutions for expressing the beamforming vector as a multicast beamformer, each EIC requires exhaustive searching. However, the favorable aspect is that the number of feasible EICs is finite in the case of clique cover index coding. Consequently, a finite search over the feasible EICs can lead to the solution for problem $(\mathsf{P1})$ given cached information $\mathcal{M}$.

C. Complexity Analysis

We shall briefly explore the computational complexity of our proposed schemes using big-O notation, considering the worst-case scenario. With the EIC problem transformed into multiple IC problems and each node serving as a central server during their communication rounds, the maximum count of decodable index codes in each communication round reaches up to 2^K. Given that the encodable/decodable EIC can be established via combinations of ICs over all communication rounds, its complexity is represented as $O(2^{K^2})$. For CGB approach, it's essential to compute the conjugate gradient in every iteration, a task with a complexity of $O(K^3N_t)$ [9], considering a cap of r iterations. In summary, the cumulative computational complexity of our proposed methodology becomes $O(2^{K^2}\cdot rK^3N_t)$.

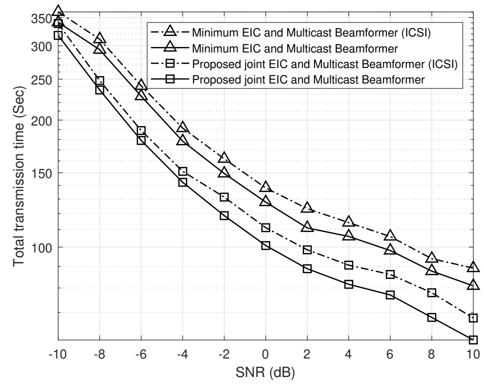

Fig. 1. The total transmission time with respect to the transmit SNR.

IV. Numerical Result

In this section, we evaluate the performance of our proposed index coding and beamforming design by assessing the total transmission time. We conduct simulations by randomly providing cached information for all users in the simulation environment and averaging the results over 50 random scenarios. We set $K=5, B=10^5$.

As shown in Fig. 1, our proposed joint design of EIC and beamformers outperforms the benchmark which represents the sequential and separate design of EIC and beamformers. These results highlight the importance of optimizing both the EIC and beamformer design together in wireless MIMO settings. Merely minimizing the length of the EIC does not guarantee optimal transmission time. Furthermore, we evaluate both schemes in the context of imperfect channel state information (ICSI), where the channel estimation error conforms to a $\mathcal{CN}(0,\sigma^2)$ distribution with $\sigma^2=10^{-3}$ [11]. In practical scenarios, attaining perfect CSI is complex, resulting in observed performance degradation as depicted in Fig. 1. It's worth highlighting that designing the multicast beamformer while accommodating imperfect CSI holds significance and will be a focal point of our forthcoming research.

V. Conclusion

In this work, we addressed the problem of joint index coding and beamforming design in a wireless MIMO broadcast channel. By considering the optimization of the total transmission time, we proposed a two-step joint design approach and compared it with a benchmark design method. The results unequivocally demonstrated the effectiveness of our proposed approach, surpassing the performance of the benchmark. This highlights that solely minimizing the length of the index coding is not always sufficient for achieving the optimal transmission time in the wireless MIMO scenario.

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